METHOD 527.2

MULTI-EXCITER TEST

CONTENTS

CONTENTS-Continued

FIGURES

METHOD 527.2 ANNEX A
ENGINEERING INFORMATION FOR MET TRANSDUCER PLACEMENT

ANNEX A FIGURE

METHOD 527.2 ANNEX B
SYSTEM IDENTIFICATION FOR LINEAR TIME-INVARIANT MDOF SYSTEMS

CONTENTS-Continued

Paragraph **Page**

METHOD 527.2 ANNEX C
PROCEDURE I MET (TIME WAVEFORM REPLICATION (TWR) SPECIFIC)

METHOD 527.2 ANNEX D
PROCEDURE II MET (SPECTRAL DENSITY MATRIX (SDM) SPECIFIC)

ANNEX D TABLES

METHOD 527.2 ANNEX E
LABORATORY VIBRATION TEST SCHEDULE DEVELOPMENT FOR MULTI-EXCITER APPLICATIONS

CONTENTS-Continued

Paragraph **Page**

METHOD 527.2

MULTI-EXCITER TEST

NOTE: **Tailoring is required**. Select methods, procedures, and parameter levels based on the tailoring process described in Part One, paragraph 4, and Annex C. Apply the general guidelines for laboratory test methods described in Part One, paragraph 5 of this standard.

Although various forms of multi-exciter test (MET) have been discussed in the technical literature and conducted in the laboratory dating back over multiple decades, there are still many issues regarding standardization of laboratory MET. In this early version of the Multi-Exciter Test Method, the intent is to introduce the basic definitions and structure of a laboratory-based multi-exciter test. MET hardware and control algorithms have continued to improve at an impressive rate recently, and MET is becoming more common in many dynamic test facilities. Feedback from the growing MET user community is highly encouraged, will be reviewed, and will play a major role in improving this Method.

Organization. The main body of this Method is arranged similarly to that of other methods of MIL-STD-810. A considerable body of supplementary information is included in the Annexes. Reference citations to external documents are at the end of the main body (paragraph 6.1). The Annexes are structured as follows:

ANNEX A - ENGINEERING INFORMATION FOR MET TRANSDUCER PLACEMENT

ANNEX B - SYSTEM IDENTIFICATION FOR LINEAR TIME INVARIANT MDOF SYSTEMS

ANNEX C - PROCEDURE I MET (TIME WAVEFORM REPLICATION (TWR) SPECIFIC)

ANNEX D - PROCEDURE II MET (SPECTRAL DENSITY MATRIX (SDM) SPECIFIC)

ANNEX E - LABORATORY VIBRATION TEST SCHEDULE DEVELOPMENT FOR MULTI-EXCITER APPLICATIONS

1. SCOPE.

1.1 Purpose.

Multi-exciter test methodology is performed to demonstrate, or provide a degree of confidence if multiple test items are considered, that materiel can structurally and functionally withstand a specified dynamic environment, e.g., stationary, non-stationary, or of a shock nature, that must be replicated on the test item in the laboratory with more than one motion degree-of-freedom. The laboratory test environment may be derived from field measurements on materiel, or may be based on an analytically-generated specification.

1.2 Application.

a. General. Use this Method for all types of materiel except as noted in Part One, paragraph 1.3, and as stated in paragraph 1.3 below. For combined environment tests, conduct the test in accordance with the applicable test documentation. However, use this Method for determination of dynamic test levels, durations, data reduction, and test procedure details.

b. Purpose of Test. The test procedures and guidance herein are adaptable to various test purposes including development, reliability, qualification, etc.

c. Dynamics Life Cycle. Table 514.8-I provides an overview of various life cycle situations during which some form of vibration (stationary or non-stationary) may be encountered, along with the anticipated platform involved.

1.2.1 General Discussion.

Use this Method to demonstrate that the materiel of interest can structurally and functionally withstand a specified dynamic environment that is defined in more than a single-degree-of-freedom (SDOF) motion; i.e., in multiple-degree-of-freedom (MDOF) motion. Establishing confidence intervals may also be of interest if multiple like items are under test. Specification of the environment may be through a detailed summary of measured field data related to the test materiel that entails more than one degree-of-freedom, or analytical generation of an environment that has been properly characterized in MDOF. In general, specification of the environment will include several degrees of freedom in a materiel measurement point configuration, and testing of the materiel in the laboratory in a SDOF mode is considered inadequate to properly distribute vibration energy in the materiel in order to satisfy the specification. As a result of the increased complexity of application of MET over multiple application of SDOF single-exciter testing (SET), an analyst, after careful review of the available data and specification, will need to provide rationale for selection of this Method. Methods 514.8, 516.8, 519.8, and 525.2 provide guidance in developing the rationale and requirement for MET.

Reasons for selection of MET over SET may include the following.

a. MET provides a distribution of vibration or shock energy to the materiel in more than one axis in a controlled manner without relying on the dynamics of the materiel for such distribution.

b. MET may be selected when the physical configuration of the materiel is such that its slenderness ratio is high, and SET must rely on the dynamics of the materiel to distribute energy.

c. For large and heavy test materiel, more than one exciter may be necessary to provide sufficient energy to the test item.

d. MET allows more degrees-of-freedom in accounting for both the impedance matches and the in service boundary conditions of the materiel.

1.2.2 Terminology.

Several terms need to be carefully defined for contrasting MET with SET. The term “test configuration” used in this document will refer to the totality of description for laboratory testing including the sources of excitation, test item fixturing, and orientation. In either testing configuration, distinction must be made between excitation measurement in a vector axis of excitation, and measurement on the test item in either the vector axis of excitation or in another vector different from the vector axis of excitation. Generally, to avoid confusion in specification and reporting, the vector directions of excitation and measurement must be specified in terms of a single laboratory inertial frame of reference related to the test configuration. In addition, it is helpful to specify the test item geometrical configuration along with the dynamic properties such as mass moments of inertia relative to the single laboratory inertial frame of reference.

a. **Single-Degree-of-Freedom (SDOF)** – motion defined by materiel movement along or about a single axis whose description requires only one coordinate to completely define the position of the item at any instant.

b. **Multi-Degree-of-Freedom (MDOF) –** motion defined by test item movement along or about more than one axis whose description requires two or more coordinates to completely define the position of the item at any instant.

c. **Single-Axis (SA) -** excitation or response measurement in a unique single vector direction (linear or rotational). For rotational axis, the vector direction is perpendicular to the plane of rotation of the exciter or test item. Figure 527.2-1 displays a single-axis input in the vertical direction to an extended structure.

d. **Multi-Axis (MA) –** excitation or response measurement that requires more than one unique vector for description. Refer to Figures 527.2-2 and 527.2-3 for MA examples of both two-axis and three-axis inputs to a common structure.

e. **Single-Exciter/Single-Axis (SESA) -** application of a single exciter providing dynamic input to the test item in a single vector direction. All SET configurations are SESA by definition.

f. **Multi-Exciter/Single-Axis (MESA) –** application of multiple exciters providing dynamic input to the test item in a single vector direction. For example, extended materiel might require excitation at the forward and aft end in a single vector axis as illustrated in Figure 527.2-2. If the definition of excitation requires more than a single vector, refer to the MEMA definition.

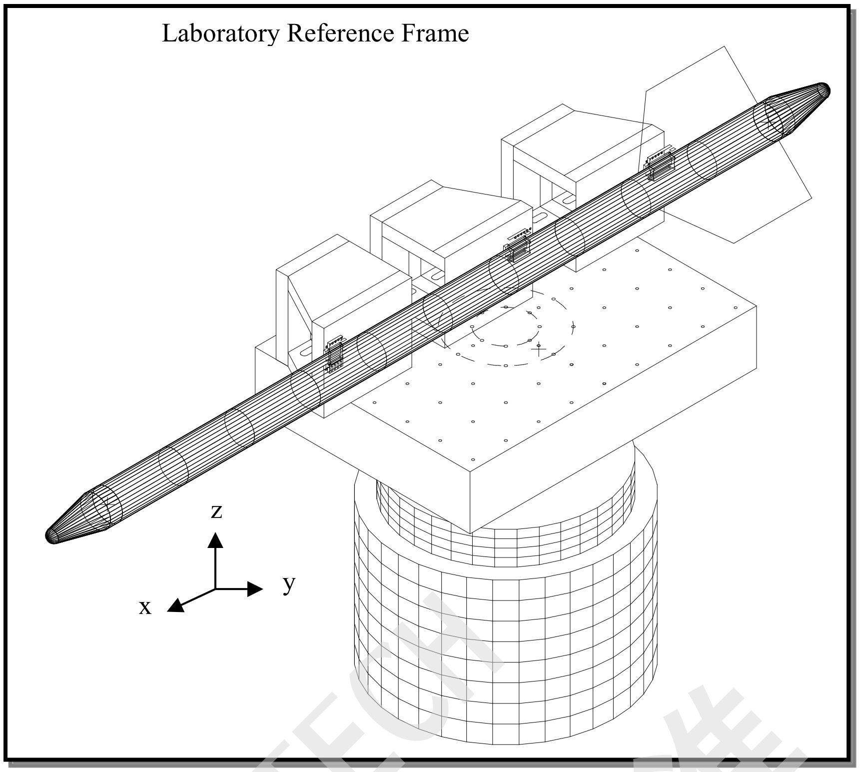


Figure 527.2-1. SESA - Single exciter vertical axis test setup.

Figure 527.2-2 illustrates a two-exciter application. Note that the system would require appropriate bearing assemblies to allow a pure rotational MESA or combined linear and rotational MEMA motion.

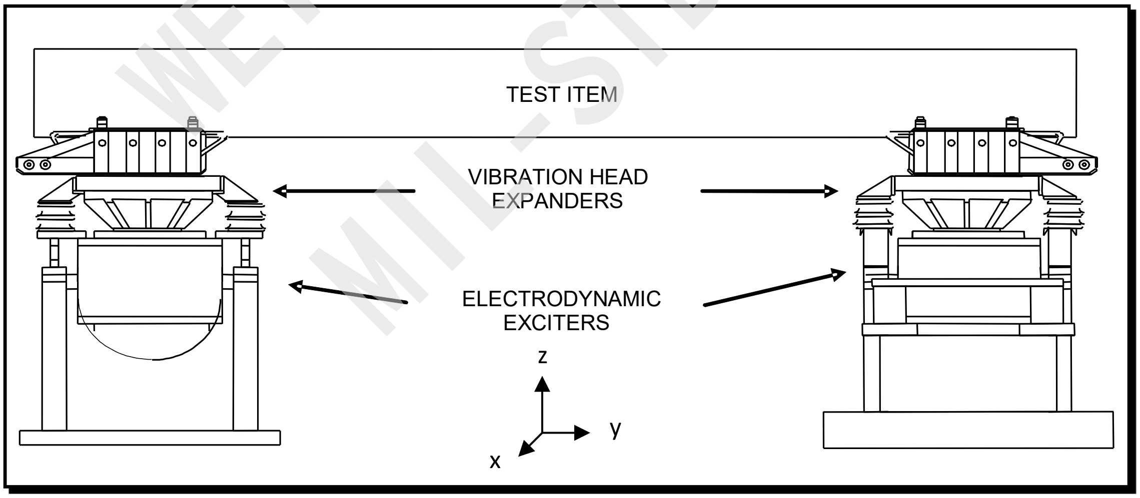


Figure 527.2-2. MESA (if control configured for two exciter 1-DOF motion) or MEMA (if control and mechanical couplings configured for two exciter 2-DOF motion).

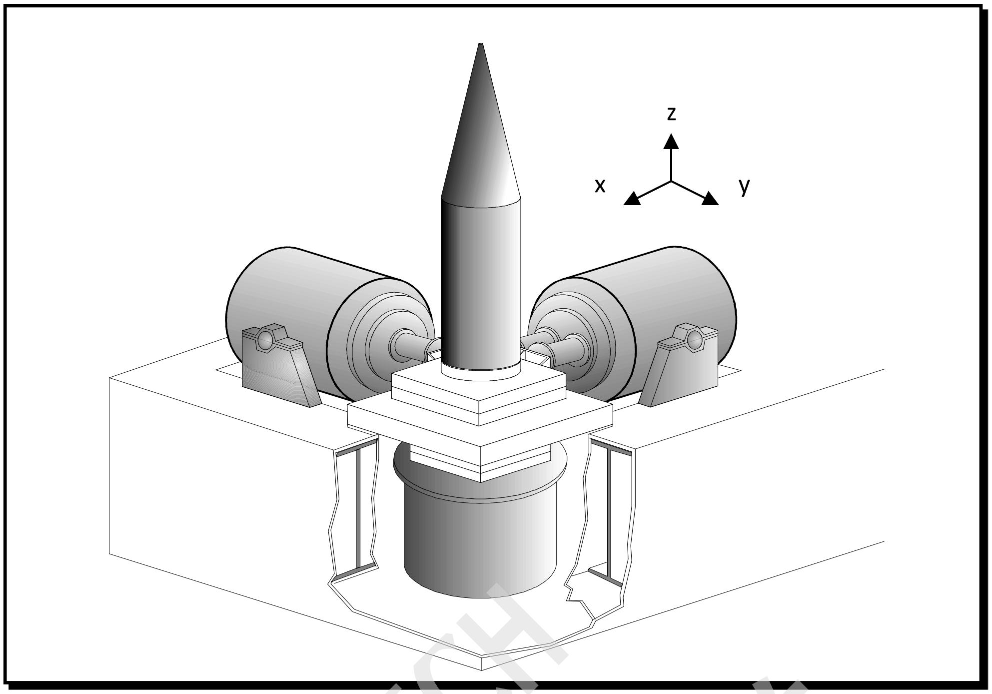


Figure 527.2-3. MEMA - Tri-axial exciter test setup (Translational Degrees-of-Freedom).

g. **Multi-Exciter/Multi-Axis (MEMA)** - Application of multiple exciters providing dynamic input to the test item in a way that requires more than a single vector for complete description of excitation and measurement. Figure 527.2-3 displays a three exciter three axis test. Three axes vertical, transverse, and longitudinal are required to describe the test. Note that many multi-axis test platform configurations have been built in recent years. Common 6 exciter examples are the hexapod (Stewart Platform), MAST, and Team Cube. There are also over-determined actuated systems consisting of more than 6 exciters. In each case, the dynamic properties vary between designs, and must be considered in the design of a MET.

h. **Single-Input/Single-Output (SISO) -** refers to input of a single drive signal to an exciter system in an SDOF configuration and a single measured output from the fixture or test item in an SDOF configuration.

i. **Single-Input/Multiple-Output (SIMO) -** refers to input of a single drive signal to an exciter system in a SDOF configuration, and multiple measured outputs from the fixture or test item in a MDOF configuration. In general, for specification purposes the dynamic behavior of the test item will not be assumed to contribute to the output DOF, i.e., measured rotation of an extended test item that is being excited in a cantilever mode will still basically be considered as a SET with linear acceleration characterizing the output.

j. **Multiple-Input/Single-Output (MISO) -** refers to input of a multiple drive signals to an exciter system configuration in a MDOF configuration, and a single measured output from the fixture or test item in a SDOF configuration. This terminology is most used in measurement data processing where the single output is a composite of measurements from multiple inputs.

k. **Multiple-Input/Multiple-Output (MIMO) -** refers to input of multiple drive signals to an exciter system configuration in a MDOF configuration, and multiple measured outputs from the fixture or test item in a MDOF configuration. It is important to note that generally there is no one-to-one correspondence between inputs and outputs, and the number of inputs and number of outputs may be different.

In the paragraphs to follow, generally only the terms MESA and MEMA will be used, however, for processing measurement data the terms SISO, SIMO, MISO, and MIMO are standard (paragraph 6.1, references a and c).

1.3 Limitations.

This Method addresses very general testing configurations for applying excitation in multiple axes to materiel. Generally, field deployed materiel has boundary (or impedance) conditions that are very difficult and often cost prohibitive to replicate in laboratory testing. The overall goal of a MET is to achieve a distribution of materiel excitation energy that approaches that appearing during in-service deployment, while minimizing the difference between in-service and laboratory boundary conditions. Fixturing design limitations and/or other physical constraints may limit application of in-service environment in the laboratory. Also, in-service measurements may not be adequate to specify the laboratory test configuration. As always, engineering analysis and judgment will be required to ensure the test fidelity is sufficient to meet the test objectives.

The following limitations also apply to this Method:

a. It does not address aspects of vendor-supplied software control strategy for a MET.

b. It does not address advantages or disadvantages of Procedure I and Procedure II MET as defined in paragraph 2.2. The state of the art in a MET is not such that a comprehensive comparison can be made at this time.

c. It does not address optimization techniques of the laboratory test configuration relative to distribution of the excitation energy within the test item.

d. It does not address technical issues related to axes of excitation and materiel mass and product moments of inertia. Nor does it address the need for specialized software for optimizing the axes of excitation with respect to mass and products of inertia.

e. It generally does not provide specific test tolerance information that is highly dependent on the (1) test objective, (2) test laboratory measurement configuration, and (3) vendor software control strategy.

f. It does not discuss, in detail, the potential for efficiencies and efficacies of a MET over SET, leaving this as a part of specification of MET peculiar to the in-service measured environment.

g. It does not discuss optimum in-service measurement configuration factors consistent with a MET.

h. It assumes that excitation is provided mechanically through electro-dynamic or servo-hydraulic exciters, and does not consider combined acoustic (refer to Method 523.4) or pneumatic induced modes of excitation.

2. TAILORING GUIDANCE.

2.1 Selecting the MET Method.

After examining requirements documents and applying the tailoring process in Part One of this Standard to determine where significant excitation energy distribution effects are foreseen in the life cycle of the materiel, or substantial testing cost savings might be achieved by employing MET strategy, use the following to confirm the need for this Method, and to place it in sequence with other Methods.

2.1.1 Effects of the MET Environment.

In general, all in-service measured environments require multiple axis response measurements for complete description. Generally, a MET will distribute excitation energy to the test item and minimize the effects of in-service boundary conditions. The following is a partial list of effects to materiel that may be better replicated in the laboratory under a MET than a SET.

a. Fatigue, cracking, and rupture sensitive to multi-axis excitation.

b. Deformation of materiel structure, e.g., protruding parts.

c. Loosening of seals and connections.

d. Displacement of components.

e. Chafing of surfaces with single-axis design.

f. Contact, short-circuiting, or degradation of electrical components.

g. Misalignment of materiel components (e.g., optical).

2.1.2 Sequence Among Other Methods.

a. General. See Part One of this Standard, paragraph 5.5.

b. Unique to this Method. Generally, a MET-specified environment may occur at any time during the life cycle of the materiel, and may be interspersed among specially designed multiple axis SET environments, e.g., shock. Perform tests representing critical end-of-mission environments last. For most tests, this can be varied if necessary to accommodate test facility schedules, or for other practical reasons.

2.2 Selecting a Procedure.

Two basic test procedures are defined under MET. The MESA or MEMA procedures may be used in replication of either a field measured materiel response or an analytically prescribed multi-axis environment. The two basic test procedures are summarized as follows:

a. Procedure I – Time Domain Reference Criteria. This MET Procedure is an extension to the SESA Time Waveform Replication (TWR) techniques addressed in Method 525.2. As with the case for SESA, the time histories measured or synthesized for a MEMA TWR test are not limited to stationary Gaussian structures.

b. Procedure II – Frequency Domain Reference Criteria. This MET Procedure is an extension to the SESA Spectral based vibration control techniques addressed in Method 514.8. As with the case for SESA, the time histories synthesized for a MEMA random test will be stationary and Gaussian in structure.

2.2.1 Procedure Selection Considerations.

Based on the test data requirements, determine if this Method is applicable. In particular, determine if there is carefully measured and properly processed materiel field measurement configuration information available in the form of band-limited time histories or auto- and cross-spectral density estimates as appropriate to be consistent with the laboratory MET configuration and vibration control system vendor software specification requirements. Basic consideration is given to an environment in a single-axis requiring multiple exciters, or an environment in multiple axes requiring multiple exciters. Generally, the MEMA procedure exceeds the complexity of the MESA procedure, so attempts should be made to minimize the test procedure complexity to the degree possible.

Materiel in-service use, along with significant environment energy distribution effects, should assist in procedure selection. One major consideration, in selection of Procedure I, is the ability to address scenarios in which the reference signal statistics are not stationary and Gaussian. Procedure II should be considered in the event that the reference data are stationary, and the ensemble of signals representing the service life may be reasonably represented by a Gaussian probability density function, and/or when time compression techniques are to be employed. Refer to the guidance provided in paragraph 4.2.2.1 of Method 514.8 regarding manipulation of kurtosis to address non-Gaussian behavior.

2.3 Determine Test Levels and Conditions.

Generally, both procedures require in-service measured response data. Procedure I will require multiple time traces to serve as the test references, and Procedure II will require the measured data to have been processed into auto- and cross-spectral density estimates in determining test levels and conditions. However, it is also possible that a MET procedure may rely on analytically specified time histories or auto- and cross-spectral density information.

2.3.1 Laboratory Test Data Input.

Acceptable engineering practice as described in paragraph 6.1, reference e, should be used to provide in-service materiel response measurement data that may be used directly in specifying one of the procedures for a MET, or may be inferred as representative of an environment that may be indirectly specified for one of the procedures for a MET. In either direct or indirect use of measurements, particular measurements are made relatively independent of materiel structure or in “zones” of the materiel that are insensitive to local conditions. It is also assumed that in-service, materiel response measurements correspond with materiel response measurements to be made in the laboratory under a MET. It is essential that the mass properties of the materiel be determined, including center-of-gravity and the mass and product moments of inertia. Whenever practical, obtain a modal survey of both the in-service and the laboratory materiel configurations. This will allow assessment of the overall dynamic characteristics of the two configurations, in addition to identifying any non-linearities as a result of materiel joints, etc. Proper interpretation of the normal mode analysis will assist in determining an optimum laboratory test configuration based on in-service measurements. Even a simple mass/stiffness analytical model will greatly assist in establishing an optimum laboratory test configuration. Give careful attention to the form and nature of the input information into the MET vendor supplied software.

2.3.1.1 Cross-Spectral Density Considerations.

In the conduct of a MET, the definition of the cross-spectral density (CSD) terms play a major role in the degree to which the characteristics of the laboratory motion correlates to the field measurements in terms of both joint spectral and temporal characteristics. In the case of Procedure I (time domain reference) the CSD information is preserved within the individual time histories to be used as reference criteria. In the case of Procedure II (frequency domain reference) the CSD terms need to be specified based on CSD estimates computed from field data. Annex D addresses the control of CSD terms in more detail.

2.3.1.2 General.

Identify the test conditions, particularly with respect to temperature conditions. Exercise extreme care in consideration of the details in the tailoring process. Base these selections on the requirements documents, the Life Cycle Environmental Profile, and information provided with this procedure.

2.3.2 Laboratory Test Output.

In addition to the considerations in paragraph 2.3.1, the test item may be instrumented at locations other than the points of MET "control," and these points are generally termed per discussion in paragraph 2.3.1 "monitoring" points. Such measurement points may be useful for other purposes such as analytical modeling of materiel and materiel components. Such measurement information and its use will not be discussed further here.

2.4 Test Item Operation.

Whenever practical, ensure test items are active and functioning during vibration tests. Monitor and record achieved performance. Obtain as much data as possible that defines the sensitivity of the materiel to vibration. Where tests are conducted to determine operational capability while exposed to the environment, operate the test item. In other cases, operate the test item where practical. Operation during transportation will not be possible in almost all cases. Also, there are cases where the operational configuration varies with mission phase, or where operation at high levels of vibration may not be required, and may be likely to result in damage.

3. INFORMATION REQUIRED.

The following minimal information is required to conduct and document dynamic tests adequately. Tailor the lists to the specific circumstances, adding or deleting items as necessary. Performing fixture and materiel modal surveys is highly recommended. These data are useful in evaluating test results, and in evaluating the suitability of materiel against changing requirements or for new applications. These data can be particularly valuable in future programs where the major emphasis will be to use existing materiel in new applications. (When modal survey is ruled out for programmatic reasons, a simple resonance search can sometimes provide useful information).

3.1 Pretest.

The following information is required to adequately conduct a MET.

a. General. Information listed in Part One, paragraphs 5.7 and 5.9 of this Standard, and in Part One, Annex A, Task 405 of this Standard.

b. Specific to this Method.

    (1) Selection of test procedure and test system (test item/platform configuration) detailed information including:

        (a) Control sensor locations for control time traces (refer to Annex A for MET specific considerations).

        (b) Reference time histories for a Procedure I MET, or reference ASD & CSD for a Procedure II MET.

        (c) Monitor/limit sensor locations (if any).

        (d) Levels of pre-test acceptable to obtain appropriate shaker system compensation.

(e) Criteria for satisfaction of the test, including previously agreed MET tolerance limits.

(2) Ability of overall system to replicate either a measured materiel environment or an analytically specified materiel environment under a MET, including bandlimited input and the temperature effects (if any).

c. Tailoring - Necessary variations in the basic test parameters/testing materials to accommodate Life Cycle Environmental Profile (LCEP) requirements and/or facility limitations.

3.2 During Test.

Collect the following information while conducting the test.

a. General. Information listed in Part One, paragraph 5.10, and in Annex A, Tasks 405 and 406 of this Standard.

b. Specific to this Method.

(1) Capture of the appropriately processed control time trace information in digital form for comparison with the specification. Compute key time domain engineering unit (EU) specific metrics such as rms versus time and key spectral metrics such as auto-spectral and cross-spectral density estimates, and ensure compliance with agreed-upon tolerances.

(2) Capture of the appropriately processed monitor/limit time trace information in digital form.

(3) Recording of the number of exposures and the duration of the dynamic environments.

(4) Log of auxiliary environmental conditions such as temperature.

(5) Log of any out of tolerance conditions relative to the control measurement points.

(6) Log of materiel functional failure.

3.3 Post-Test.

The following post-test data shall be included in the test report.

a. General. Information listed in Part One paragraph 5.13, and in Annex A, Task 406 of this Standard.

b. Specific to this Method.

(1) Duration of exposure of the materiel to the dynamic MET environment.

(2) Any data measurement anomalies, e.g., high instrumentation noise levels, loss of sensor response.

(3) Status of the test materiel/fixture. In particular, any structural or functional failure of the test materiel/fixture.

(4) Status of measurement system after each test.

(5) Any changes from the original test plan.

4. TEST PROCESS.

Tailor the following sections as appropriate for the individual contract or program.

4.1 Test Facility.

The specialized nature of a MET requires use of a test facility that includes proven MET capability, fixture(s) for mounting the test materiel, and appropriate equipment for recording the response of the test item at the specified control and monitor locations. In addition, the test facility will have expertise necessary to appropriately configure the test according to the form of test materiel as outlined in paragraph 2.2.1. Ensure the exciter control has appropriately validated vendor supplied MET hardware and software.

4.2 Controls.

The accuracy in providing and measuring shock and vibration environments is highly dependent on fixtures and mountings for the test item, the measurement system, and the exciter control strategy. Ensure all instrumentation considerations are in accordance with the best practices available (see paragraph 6.1, references d and e). Careful design of the test set up, fixtures, transducer mountings, and wiring, along with good quality control will be necessary to meet the tolerances of paragraph 4.2.2 below.

4.2.1 Calibration.

Ensure the excitation apparatus, all transducers, signal conditioning equipment, independent measurement systems, and the vibration control system are calibrated for conformance with the specified test requirement. Careful design of the test set up, fixtures, transducer mountings and wiring, along with good quality control will be necessary to meet the tolerances of paragraph 4.2.2 below.

4.2.2 Tolerances.

The question of reasonable tolerances in a MET is not simple for either MET procedure. Guidelines for establishing test tolerances for a Procedure I MET are discussed in Annex C, and tolerances for a Procedure II MET are discussed in Annex D. Due to the unique factors associated with a MET, test metrics will often need to be addressed on a test by test basis. It is critical that the test objectives be clearly understood prior to establishing test tolerances, and that the metrics are carefully documented prior to conduct of the test.

4.3 Test Interruption.

Test interruptions can result from multiple situations. The following paragraphs discuss common causes for test interruptions, and recommended paths forward for each. Recommend test recording equipment remain active during any test interruption if the excitation equipment is in a powered state.

4.3.1 Interruption Due To Laboratory Equipment Malfunction.

a. General. See Part One, paragraph 5.11, of this Standard.

b. Specific to this Method. When interruptions are due to failure of the laboratory equipment, analyze the failure to determine root cause. It is also strongly advised that both control and response data be evaluated to ensure that no undesired transients were imparted to the test item during the test equipment failure. If the test item was not subjected to an over-test condition as a result of the equipment failure, repair the test equipment or move to alternate test equipment and resume testing from the point of interruption. If the test item was subjected to an over-test condition as a result of the equipment failure, immediately notify the test engineer or program engineer responsible for the test item. Conduct a risk assessment based on factors such as level and duration of the over-test event, spectral content of the event, cost and availability of test resources, and analysis of test specific issues to establish the path forward. See Method 514.8, Annex A, paragraph 2.1 for descriptions of common test types, and a general discussion of test objectives.

4.3.2 Interruption Due To Test Item Operation Failure.

Failure of the test item(s) to function as required during operational checks presents a situation with several possible options. Failure of subsystems often has varying degrees of importance in evaluation of the test item. Selection of options a through c below will be test specific.

a. The preferable option is to replace the test item with a “new” item and restart the entire test.

b. An alternative is to replace/repair the failed or non-functioning component or assembly with one that functions as intended, and restart the entire test. Conduct a risk analysis prior to continuing since this option places an over-test condition on the entire test item except for the replaced component. If the non-functioning component or subsystem is a line replaceable unit (LRU) whose life-cycle is less than that of the system test being conducted, proceed as would be done in the field by substituting the LRU, and continue from the point of interruption.

c. For many system level tests involving either very expensive or unique test items, it may not be possible to acquire additional hardware for re-test based on a single subsystem failure. For such cases, a risk assessment should be performed by the organization responsible for the system under test to determine if replacement of the failed subsystem and resumption of the test is an acceptable option. If such approval is provided, the failed component should be re-tested at the subcomponent level.

NOTE: When evaluating failure interruptions, consider prior testing on the same test item and any consequences of such.

4.3.3 Interruption Due To A Scheduled Event.

There are often situations in which scheduled test interruptions will take place. For example, in a tactical transportation scenario, the payload may be re-secured to the transport vehicle periodically (i.e., tie-down straps may be re-secured at the beginning of each day). Endurance testing often represents a lifetime of exposure; therefore it is not realistic to expect the payload to go through the entire test sequence without re-securing the tie-downs as is done in a tactical deployment. Many other such interruptions, to include scheduled maintenance events, are often required over the life-cycle of materiel. Given the cumulative nature of fatigue imparted by dynamic testing, it is acceptable to have test interruptions that are correlated to realistic life-cycle events. All scheduled interruptions should be documented in the test plan and test report.

4.3.4 Interruption Due To Exceeding Test Tolerances.

Exceeding the test tolerances defined in paragraph 4.2.2, or a noticeable change in dynamic response may result in a manual operator initiated test interruption or an automatic interruption when the tolerances are integrated into the control strategy. In such cases, check the test item, fixturing, and instrumentation to isolate the cause.

a. If the interruption resulted from a fixturing or instrumentation issue, correct the problem and resume the test.

b. If the interruption resulted from a structural or mechanical degradation of the test item, the problem will generally result in a test failure, and a requirement to re-test unless the problem is allowed to be corrected during testing. If the test item does not operate satisfactorily, follow the guidance in paragraph 4.3.2 for test item failure.

4.4 Test Setup.

4.4.1 Instrumentation.

Various sensor types can be used in a MET setup and used to establish the need for a MET. In general, and used in examples throughout this document, acceleration will be the quantity measured to establish the specification for the procedure. Processed sensor measurement information from the lab environment should correspond to processed measurement information made in the field. This is ideally accomplished by mounting the test item accelerometer in the same location as that on the field measurement materiel from which the measured information was extracted. In the MDOF case, instrumentation location and polarity become critical test parameters (refer to Annex A). To maintain proper phase relationships between channels, a synchronous sample and hold analog to digital converter (A/D) is recommended. When possible, recommend laboratory and field data acquisition and instrumentation be the same. Otherwise, it may be necessary to precondition reference data prior to conduct of a laboratory test.

Calibrate all measurement instrumentation to traceable national calibration standards (see Part One, paragraph 5.3.2). The measurement device and its mounting will be compatible with the requirements and guidelines provided in paragraph 6.1, reference e.

a. Accelerometer. In the selection of any transducer, one should be familiar with all parameters provided on the associated specification sheet. Key performance parameters for an accelerometer follow:

    (1) Frequency Response: A flat frequency response within ± 5 percent across the frequency range of interest is required.

    (2) Transverse sensitivity should be less than or equal to 5 percent.

    (3) Nearly all transducers are affected by high and low temperatures. Understand and compensate for temperature sensitivity deviation as required. Temperature sensitivity deviations at the test temperature of interest should be no more than ±5% relative to the temperature at which the transducer sensitivity was established.

    (4) Base Strain sensitivity should be evaluated in the selection of any accelerometer. Establishing limitations on base strain sensitivity is often case specific based upon the ratio of base strain to anticipated translational acceleration.

(5) High sensitivity accelerometers are recommended when linear accelerometers are employed to make rotational motion estimates.

b. Other measurement devices. Any other measurement devices used to collect data must be demonstrated to be consistent with the requirements of the test.

4.4.2 Platform Integration.

a. Test Fixture Design. Observe standard shock and vibration fixture design practices with regard to frequency response and the ability to withstand the reaction forces with consideration of potentially high loads generated during MEMA tests as a result of the accelerations applied simultaneously in multiple degrees of freedom.

b. Test Configuration. Both MESA and MEMA tests require that the test configuration be restrained in all degrees of freedom that are not controlled by the exciter, and released in all degrees of freedom that are. A kinematic assessment of the setup is recommended to assist in the selection of the proper couplings, bearings, etc., to ensure that improper loads are not transferred to the test item through the controlled application of the test, as well as the potentially uncontrolled motion of the exciters.

4.4.3 Setup Analysis

In general, because of impedance mismatches and boundary condition effects, differences between the field and laboratory environments will exist. Such differences between the laboratory measured and test specified information may require further analysis with respect to the original field data and payload dynamics to determine if the differences are relevant to the test objectives.

a. Rudimentary analysis to ensure the test tolerances are met is usually performed within the MET software and control strategy. Laboratory personnel should consult the vendor-supplied MET control system documentation, and clearly understand the determination of these test tolerances. In most cases this will require direct contact with the vendor of the MET system. At the time of this initial publication, common examples of analysis techniques that are performed during a MET include computation of EU-rms versus time, ASD, CSD, peak-detection, and histograms.

b. More extensive data analysis can be performed to examine the significance of test tolerance deviations with off-line specialized software. Refer to Method 525.2, Annex B for Procedure I analysis methods, and paragraph 6.1, references d and e for a variety of detailed analysis techniques for random data applicable for Procedures I and II.

4.5 Test Execution.

4.5.1 Preparation for Test.

Carefully examine the reference time histories or specified auto- and cross-spectral information for validity. Ensure the test specification is band-limited according to the band limits of the shaker system. In particular, it may be necessary to remove any high amplitude low frequency components that will cause an over-travel condition for the shaker control system or result in velocity limit violation. In the event the reference data must be modified to address exciter system limitations, care must be exercised to ensure the intent of the test is not compromised; and the modifications must be documented and approved by the responsible test officer. Most MET systems do provide for such exciter system limit checks; however, the feasibility of exciter reproduction relative to cross-spectral information is generally not checked.

Characterize the materiel to be tested. For example:

a. Dynamically flexible structure with a varying length/diameter ratio.

b. Dynamically stiff structure with flexible appendages.

c. Dynamically/geometrically asymmetric structure.

d. Materiel in shipping or storage containers with pursuant materiel/container isolation.

If the test item is unique and must not be degraded before laboratory testing, test a dynamic simulation item that represents the dynamic properties of the materiel to be tested to ensure the MET can be properly compensated. Such a preliminary test will allow specification and refinement of the control strategy, including selection of control measurement points. It may also allow specification of the overall exciter configuration for optimizing the test strategy.

4.5.1.1 Preliminary Steps.

Before starting a test, review pretest information in the test plan to determine test details (procedure(s), test item configuration(s), levels, durations, vibration exciter control strategy, failure criteria, test item operational requirements, instrumentation requirements, facility capability, fixture(s), etc.).

a. Select the appropriate MET configuration and associated fixturing.

b. Select the appropriate data acquisition system (e.g., instrumentation, cables, signal conditioning, recording, and analysis equipment).

c. Operate vibration equipment without the test item installed to confirm proper operation.

d. Ensure the data acquisition system functions as required.

4.5.1.2 Pretest Standard Ambient Checkout.

All items require a pretest standard ambient checkout to provide baseline data. Conduct the pretest checkout as follows:

Step 1 Examine the test item for physical defects, etc., and document the results.

Step 2 Prepare the test item for test, in its operating configuration if required, as specified in the test plan.

Step 3 Examine the test item/fixture/excitation system combination for compliance with test item and test plan requirements.

Step 4 If applicable, conduct an operational checkout in accordance with the test plan and document the results for comparison with data taken during or after the test. If the test item fails to operate as required, resolve the problems and repeat the operational checkout.

4.5.2 Procedure.

The following steps provide the basis for collecting the necessary information concerning the platform and test item under MET testing.

a. Procedure I – Time Domain Reference Criteria.

Step 1 Select the test conditions to be addressed and mount the test item on the excitation platform. Select the control locations and associated analysis techniques that will be used as potential test metrics (refer to Method 525.2, Annex A, and Annexes A, B, and C of this Method). Placement and polarity of all sensors (i.e. accelerometers) must match that of the reference signals (refer to Annex A). Clearly identify each axis of excitation and provide alignment procedures to ensure all measurements are made precisely along each excitation axis. Use all inherent information concerning the dynamic/geometric configuration of the test item, including specification of the center-of-gravity of the test item in three orthogonal axes, modal characteristics of the test fixturing, and all pertinent mass moments of inertia.

Step 2 If required; perform an operational check of the test item at defined environmental test conditions per the test plan. If the test item operates satisfactorily, proceed to Step 3. If not, resolve the problem(s) and repeat this step.

Step 3 Subject the test item (or dynamic simulant) to a system identification process that determines the initial exciter drive voltage signals by compensation. For the MDOF case, the initial signals sent to the exciters for compensation must be statistically independent, and form vectors that are linearly independent with respect to the DOFs to be tested. If a dynamic simulant is used, replace the dynamic simulant with the test item subsequent to the system identification and compensation phase.

Step 4 Subject the test item in its operational mode to the TWR compensated waveform. It is often desirable to make an initial run at less than full level to ensure proper dynamic response, and to validate proper functioning of the instrumentation.

Step 5 Record necessary data, including the control sensor time traces that can be processed to demonstrate that satisfactory replication of the matrix of reference time trace signals has been obtained.

Step 6 Continuously monitor vibration levels and, if applicable, test item performance throughout the exposure. If levels shift or a failure occurs, shut down the test in accordance with the test interruption procedure (paragraph 4.3.2). Determine the reason for the shift and proceed in accordance with the test interruption recovery procedure (paragraph 4.3.2).

Step 7 Repeat Steps 4, 5, and 6 as specified in the test plan.

Step 8 Remove the test item from the fixture and perform an operational check. Inspect the test item, mounting hardware, packaging, etc., for any signs of visual mechanical degradation that may have occurred during testing. See paragraph 5 for analysis of results.

b. Procedure II – Frequency Domain Reference Criteria.

Step 1 Select the test conditions to be addressed and mount the test item on the excitation platform. Select the control locations and associated analysis techniques that will be used as potential test metrics (refer to Annexes A, B, and D of this Method). Placement and polarity of all sensors (i.e. accelerometers) must match that of the reference signals (refer to Annex A). Clearly identify each axis of excitation and provide alignment procedures to ensure all measurements are made precisely along each excitation axis. Use all inherent information concerning the dynamic/geometric configuration of the test item, including specification of the center-of-gravity of the test item in three orthogonal axes, modal characteristics of the test fixturing, and all pertinent mass moments of inertia.

Step 2 If required; perform an operational check on the test item at defined environmental test conditions per the test plan. If the test item operates satisfactorily, proceed to Step 3. If not, resolve the problem(s) and repeat this step.

Step 3 Subject the test item (or dynamically accurate surrogate if available) to a system identification process. For the MDOF case, the initial signals sent to the exciters must be statistically independent and form vectors that are linearly independent with respect to the DOFs to be tested. If a dynamic simulant is used, replace the dynamic simulant with the test item subsequent to the system identification and compensation phase.

Step 4 Subject the test item in its operational mode to the specification levels, monitoring both auto and cross-spectral density terms. It is almost always necessary to make an initial run at less than full level to ensure proper dynamic response, and to validate proper functioning of the instrumentation.

Step 5 Record necessary data, including the control sensor auto and cross-spectral estimates that demonstrate satisfaction of the overall test objectives.

Step 6 Continuously monitor vibration levels and, if applicable, test item performance throughout the exposure. If levels shift or a failure occurs, determine the reason for the shift, and follow the test interruption procedure (paragraph 4.3.2).

Step 7 Repeat Steps 4, 5, and 6 as specified in the test plan.

Step 8 Remove the test item from the fixture and perform an operational check. Inspect the test item, mounting hardware, packaging, etc., for any signs of visual mechanical degradation that may have occurred during testing**.** See paragraph 5 for analysis of results.

5. ANALYSIS OF RESULTS.

In addition to the guidance provided in Part One, paragraphs 5.14 and 5.17, and Part One, Annex A, Tasks 405 and 406, the following information is provided to assist in the evaluation of the test results. Analyze in detail any failure of a test item to meet the requirements of the system specification, and consider related information such as:

a. Proper collection of information from the control accelerometer configuration, including representative durations of time trace information at all test levels based on expressions for estimate statistical error criteria. All time trace measurement information must be time-correlated to ensure proper estimation.

b. Proper collection of information from the monitor accelerometer configuration (if any), including representative durations of time trace information at all test levels according to the same principles as used for control measurements.

c. Record the vendor MET software test tolerance information.

d. If necessary, apply one or more of the techniques described in Annexes C and D for detailed comparison of the frequency domain information. In particular, use the collected time trace information to compute the agreed-upon test metrics.

5.1 Physics of Failure.

Analyses of vibration related failures must relate the failure mechanism to the dynamics of the failed item and to the dynamic environment. It is insufficient to determine that something broke due to high cycle fatigue or wear. It is necessary to relate the failure to the dynamic response of the materiel to the dynamic environment. The scope and detail of analysis should be coordinated with and approved by the appropriate test authority. It is recommended to include in the failure analysis a determination of resonant mode shapes, frequencies, damping values, and dynamic strain distributions, in addition to the usual material properties, crack initiation locations, etc.

5.2 Qualification Tests.

When a test is intended to show formal compliance with contract requirements, recommend the following definitions:

a. Failure definition. "Materiel is deemed to have failed if it suffers permanent deformation or fracture; if any fixed part or assembly loosens; if any moving or movable part of an assembly becomes free or sluggish in operation; if any movable part or control shifts in setting, position, or adjustment, and if test item performance does not meet specification requirements while exposed to functional levels and following endurance tests." Ensure this statement is accompanied by references to appropriate specifications, drawings, and inspection methods.

b. Test completion. A vibration qualification test is complete when all elements of the test item have successfully passed a complete test.

5.3 Other Tests.

For tests other than qualification tests, prepare success and/or failure criteria and test completion criteria that reflect the purpose of the tests.

6. REFERENCE/RELATED DOCUMENTS.

6.1 Referenced Documents.

a. NATO STANAG 4370, Environmental Testing, Allied Environmental Conditions and Test Publication (AECTP) 200, Mechanical Environmental Testing, Category 240.

b. International Test Operations Procedure (ITOP) 01-1-050, "Development of Laboratory Vibration Test Schedules", 6 June 1997, DTIC AD No. B227368.

c. International Test Operations Procedure (ITOP) 01-2-601, "Laboratory Vibration Schedules", 25 January 1999. DTIC AD No. B238288.

d. Bendat, Julius S. and Allan G. Piersol, Random Data Analysis and Measurement Procedures, 3rd Edition, John Wiley & Sons, Inc., New York, 2000.

e. Handbook for Dynamic Data Acquisition and Analysis, IEST-RD-DTE012.2; Institute of Environmental Sciences and Technology, Arlington Place One, 2340 S. Arlington Heights Road, Suite 100, Arlington Heights, IL 60005-4516; Institute of Environmental Sciences and Technology Website.

f. Fitz-Coy, Norman and Hale, Michael T., "On the Use of Linear Accelerometers in Six-DOF Laboratory Motion Replication: A Unified Time-Domain Analysis", Proceedings of the 76th Shock and Vibration Symposium, Nov. 2005. Shock & Vibration Exchange (SAVE), 1104 Arvon Road, Arvonia, VA 23004.

g. Underwood, Marcos A. and Keller, Tony, "Applying Coordinate Transformations to Multi-DOF Shaker Control", Sound and Vibration, January 2006, Sound and Vibration Website.

h. Shock and Vibration Handbook, 5th Edition, Edited by Cyril M. Harris and Allan G. Piersol, McGraw-Hill, New York NY, 2002.

i. Smallwood, David O., "Multiple Shaker Random Vibration Control – An Update", SAND 98-2044C.

j. Smallwood, David, Multiple-Input Multiple-Output (MIMO) Linear Systems Extreme Inputs/Outputs. Shock and Vibration 14 (2007) 107-131; IOS Press, Inc.

k. Hale, Michael T., “Consideration of Global Error Metrics in the Conduct of MDOF Motion Replication”, Proceedings of the 77th Shock and Vibration Symposium, Nov. 2006; Shock & Vibration Exchange (SAVE), 1104 Arvon Road, Arvonia, VA 23004.

l. Underwood, Marcos A. and Keller, Tony, “Using the Spectral Density Matrix to Determine Ordinary, Partial, and Multiple Coherence”, Proceedings of the 77th Shock & Vibration Symposium, October, 2006; Monterey, California.

m. Underwood, Marcos A., “Multi-Exciter Testing Applications: Theory and Practice”, Proceedings – Institute of Environmental Sciences and Technology, April 2002.

n. Plummer, A.R., “Control Techniques for Structural Testing: A Review”, Proc. IMechE Vol. 221 Part I: J. Systems and Control Engineering, 2007.

o. Welch, P.D., “The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms”, *IEEE Transactions on Audio and Electroacoustics*, Vol. AU-15, No. 2, June 1967.

p. Fitz-Coy, N, Hale, M. and Nagabhushan, V., “Benefits and Challenges of Over-Actuated Excitation Systems”, Shock and Vibration Journal, Volume 17, Number 3 / 2010.

q. Hale, Michael T., “Spectral Density Matrix Transformations”, Journal of the IEST, V. 60, No. 1, pp 17-26, 2017.

6.2 Related Documents.

Egbert, Herbert W. “The History and Rationale of MIL-STD-810 (Edition 2)”, January 2010; Institute of Environmental Sciences and Technology, Arlington Place One, 2340 S. Arlington Heights Road, Suite 100, Arlington Heights, IL 60005-4516.

(Copies of Department of Defense Specifications, Standards, and Handbooks, and International Standardization Agreements are available online at https://assist.dla.mil.

Requests for other defense-related technical publications may be directed to the Defense Technical Information Center (DTIC), ATTN: DTIC-BR, Suite 0944, 8725 John J. Kingman Road, Fort Belvoir VA 22060-6218, 1-800-225-3842 (Assistance--selection 3, option 2), http://www.dtic.mil/dtic/; and the National Technical Information Service (NTIS), Springfield VA 22161, 1-800-553-NTIS (6847), http://www.ntis.gov/.

(This page is intentionally blank.)

METHOD 527.2, ANNEX A

ENGINEERING INFORMATION FOR MET TRANSDUCER PLACEMENT

1. GENERAL PHILOSOPHY FOR A MET.

The general philosophy for a Multi-Exciter Test (MET) is essentially the same as that of the Single Exciter case; however, there are additional considerations that need to be addressed in the conduct of a MET. It is addressing the additional considerations associated with MESA and MEMA, and assessing the adequacy of a laboratory MET, i.e., comparing the reference time histories or spectral content with the results obtained in laboratory based tests, that are the concerns of this Annex. As of the inclusion of this new test method into MIL-STD-810G, the primary vibration control system vendors offer MET options for time waveform replication (TWR), sine, shock, and random. Options for combined environments such as narrowband-random-on-random and sine-on-random are generally implemented via TWR based techniques.

In the simplest terms for MESA and MEMA tests, multiple exciters are employed to excite one or more mechanical-degrees-of-freedom. For traditional SESA testing, the test reference is provided as either a single reference time trace as discussed in Method 525.2, or in terms of simple magnitude versus frequency plots such as an auto spectral density as discussed in Method 514.8. For a MET, multiple channels are required in the control process. For a MET defined in the time domain, multiple time traces will be required, and for a MET defined in the frequency domain, cross spectral densities are required in addition to auto-spectral parameters in defining the test reference. For either case, the system identification (transfer function) estimation process is now a matrix operation as opposed to a simple division as in the SESA case.

The additional complexities associated with MESA and MEMA testing require an increased level of technical skill from the test engineers in planning such tests, and from the test operators that will ultimately perform the tests. Test objectives must be clearly understood to ensure that, in addressing the inevitable test-specific obstacles associated with any MDOF test, the test objectives are still properly addressed.

2. REFERENCE POINT CONSIDERATIONS FOR MDOF TESTING.

2.1 Reference Data Considerations.

The first step in performing a MET in the laboratory begins with acquiring sufficient reference data. In addition to the standard concerns related to the dynamic range and frequency response characteristics of the transducers and recording equipment used in the field data acquisition phase, the quantity and spatial locations of the transducers become critical test parameters. Understanding the underlying dynamics of MDOF systems, and the physical constraints such systems place on the spatial locations of reference transducers in order to perform true MDOF laboratory motion replication, is not trivial. Similarly, it is essential that the test operators are able to understand the dynamics of an arbitrary data set that may be provided by an outside source for use as reference data in a laboratory test.

2.2 Reference Point Kinematics.

A unified discussion on the use of linear accelerometers for motion reconstruction is addressed in paragraph 6.1, reference f. Specifically, paragraph 6.1, reference f, investigated the number of uni-axial transducers required, and the placement of these transducers in the field data acquisition phase for 6-DOF motion reconstruction. The principal analysis is performed in the time domain using kinematical relationships from classical mechanics.

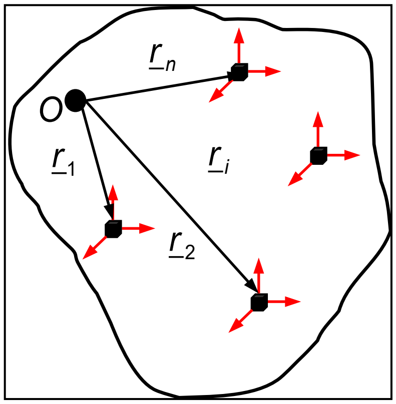


Figure 527.2A-1: Body with *n* accelerometers. Placements.

In addressing the laboratory inputs required for 6-DOF replication, paragraph 6.1, references f and p also consider a body equipped with *n* tri-axial linear accelerometers located as shown in Figure 527.2A-1. It is well known from classical mechanics that the acceleration measured by the i^{th} transducer is given kinematically by

$$\underline{a}_i = \underline{a}_O + \underline{\alpha} \times \underline{r}_i + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}_i \right) + \underline{\varepsilon}_i \,, i = 1, 2, \ldots, n, \tag{1}$$

where $\underline{a}_O$ represents the acceleration of a reference point in the body, $\underline{\alpha}$ and $\underline{\omega}$ represent, respectively, the rigid body angular acceleration and angular velocity, $\underline{r}_i$ the location of the i^{th} transducer relative to the reference point, and $\underline{\varepsilon}_i = \ddot{\underline{r}}_i + 2\underline{\omega} \times \dot{\underline{r}}_i$ represents the contributions due to non-rigid body effects (i.e., flexibility). Ignoring the flexibility effects (i.e., $\underline{\varepsilon}_i = \underline{0}$), Equation 1 represents *n* vector equations in three vector unknowns (i.e., $\underline{a}_O$, $\underline{\omega}$, and $\underline{\alpha}$). In general, $\underline{a}_O$ is unknown unless a transducer was selected *a priori* for that location. For notational convenience, matrix equivalent operations were used to rewrite Equation 1 as shown in Equation 2 where the flexibility effects have also been neglected.

$$\underline{a}_i = \underline{a}_O + \underline{\alpha}^{\times} \underline{r}_i + \underline{\omega}^{\times} \underline{\omega}^{\times} \underline{r}_i = \underline{a}_O + \underline{\underline{\Omega}}\, \underline{r}_i \,, i = 1, 2, \ldots, n \tag{2}$$

In Equation (2), $\underline{\alpha}^{\times}$ and $\underline{\omega}^{\times}$ are skew symmetric matrices representing the vector cross products, and $\underline{\underline{\Omega}} \triangleq \underline{\alpha}^{\times} + \underline{\omega}^{\times} \underline{\omega}^{\times}$ represent the contributions of angular motion to the measured linear acceleration (i.e., the contributions of "tangential" and "centripetal" accelerations). Assuming that $\underline{\alpha} = \alpha_x \hat{\underline{i}} + \alpha_y \hat{\underline{j}} + \alpha_z \hat{\underline{k}}$ and $\underline{\omega} = \omega_x \hat{\underline{i}} + \omega_y \hat{\underline{j}} + \omega_z \hat{\underline{k}}$ are the angular acceleration and angular velocity coordinatized in the body fixed frame, then

$$\underline{\alpha}^{\times} = \begin{bmatrix} 0 & -\alpha_z & \alpha_y \\ \alpha_z & 0 & -\alpha_x \\ -\alpha_y & \alpha_x & 0 \end{bmatrix}, \underline{\omega}^{\times} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}, \text{ and } \underline{\underline{\Omega}} = \begin{bmatrix} -\left(\omega_y^2 + \omega_z^2\right) & \omega_x\omega_y - \alpha_z & \omega_x\omega_z + \alpha_y \\ \omega_x\omega_y + \alpha_z & -\left(\omega_x^2 + \omega_z^2\right) & \omega_y\omega_z - \alpha_x \\ \omega_x\omega_z - \alpha_y & \omega_y\omega_z + \alpha_x & -\left(\omega_x^2 + \omega_y^2\right) \end{bmatrix}$$

True motion replication in the laboratory using the measured accelerations (field data) to construct the drive point accelerations will require knowledge of $\underline{a}_O$ (three unknowns) and $\underline{\underline{\Omega}}$ (nine unknowns), for a total of 12 unknowns. A closer examination of $\underline{\underline{\Omega}}$, however, reveals the matrix is comprised of only six unique unknowns (i.e., the components of $\underline{\alpha}$ and $\underline{\omega}$). Thus, if $\underline{a}_O$, $\underline{\alpha}$, and $\underline{\omega}$ can be determined from measured field data, theoretically, the motion in the field can be exactly (within the limits of the measurement devices) replicated in the laboratory. From paragraph 6.1, reference f, it was shown that in the most general case, nine parameters ($\underline{a}_0, \underline{\omega}, \underline{\alpha}$) are required to reconstruct the motion and, thus, the minimum number of required transducer channels is nine. The analysis was also used to show that if specific restrictions are imposed on the motion (e.g., $\underline{a}_0 = \underline{0}$), six properly placed accelerometers would be sufficient. Additionally, if consideration was given to the rigid body kinematic relationship between the angular velocity $\underline{\omega}$ and the angular acceleration $\underline{\alpha}$ (i.e., $\underline{\alpha} = \frac{d\underline{\omega}}{dt}$), then implementation in the frequency domain also reduces the number of required parameters from nine to six.

The two stated restrictions (i.e., $\underline{a}_0 = \underline{0}$ or frequency domain implementation) that result in six transducers being sufficient, are consistent with the conditions found in the vibration testing environment. An assumption of $\underline{a}_0 = \underline{0}$ does not necessarily provide sufficient information for exact motion reconstruction. In fact, it was shown that in the most general case, only $\underline{\alpha}$ could be uniquely determined and, thus, additionally, the kinematic relationship between $\underline{\alpha}$ and $\underline{\omega}$ has to be exploited. Hence, the most influential of the two restrictions is the simplified relationship between angular velocity and angular acceleration in the frequency domain (i.e., $\underline{\alpha}(s) = s\underline{\omega}(s)$). Note that this condition is only valid for rigid bodies. Once flexibility is considered, this simplification no longer exists and, thus, the use of six transducers becomes questionable.

From an implementation perspective, while it has been shown that six properly located linear accelerometers are sufficient to use as a basis for 6-DOF motion replication, it is also obvious that near ideal conditions are required. Specifically, and as is generally the case for laboratory vibration tests, $\underline{a}_0 \cong \underline{0}$ in Equation 1 is a necessary requirement to ensure accurate replication of acceleration and velocity at unmonitored points on the test item. A more realistic concern is that, in practice, one is not necessarily working with a rigid body, and the fact that there will inevitably be a mechanical impedance mismatch between the field and laboratory conditions. Under such conditions, predictably there will be issues with the condition number of the system transfer function matrix $\mathbf{H}_{\mathbf{xy}}$.

To address such issues, it is strongly recommended that an over-determined feedback scheme (number of control channels > number of mechanical DOF) consisting of properly placed linear accelerometers be employed. One such proven control configuration is selection of three non-collinear tri-axial clusters of linear accelerometers. This control configuration is very versatile in that any plane may be used, with the only critical factor being that the relative positions of the transducers remain non-collinear.

(This page is intentionally blank.)

METHOD 527.2, ANNEX B

SYSTEM IDENTIFICATION FOR LINEAR TIME-INVARIANT MDOF SYSTEMS

1. TRANSFER-FUNCTION ESTIMATIONS.

Exploiting the over-determined feedback technique as discussed in Annex A is also advantageous in providing a weighting technique, analogous to the common practice in traditional SDOF testing in which various multiple-channel averaging schemes are employed to address payload dynamics issues. In the conduct of an MDOF vibration test, if an over-determined feedback scheme consisting of properly placed linear accelerometers is employed, $\mathbf{H}_{\mathbf{xy}}$ is approximated in a Least-Squares sense, thereby providing a sound method of implementing a multi-channel control scheme. However, as is the case for the general 1-DOF case, one should always optimize the fixture design because no control scheme will force motion of a structure in an unnatural manner. The accuracy of the Least Squares approximation of $\mathbf{H}_{\mathbf{xy}}$ will be directly related to the degree of modal similarity between the field deployment platform and the laboratory test platform.

Based on the previous discussion of kinematic considerations for transducer placement, it is clear that great care must be taken to establish a central point to which all measurement locations could be referenced. Carefully measure and record the specific location and polarity of each transducer. In addition, this process requires forethought as to how the test item will be fixtured in the laboratory to ensure the “exact” measurement locations can be used.

2. SIGNAL TRANSFORMATION.

For a situation in which the reference signals for a 6-DOF test are provided in the traditional translational (X, Y, and Z) and rotational (Pitch (rotation about Y), Roll (rotation about X), and Yaw (rotation about Z)) engineering units (EU), one may wish to transform between appropriately placed linear transducers and traditional 6-DOF EUs. Since there are many combinations of exciters that may be employed for a given MDOF test, the transformation matrix between linear accelerometers and traditional 6-DOF EUs, the transformation matrix will be test specific. In addition, one may wish to apply non-uniform weighting across the exciters for a given DOF, or even include non-rotational or non-translational degrees-of-freedom such as tensional response into consideration in developing the control law for a given test. Kinematics based output-signal transformations are also very useful in addressing over-actuated systems to ensure properly compensated signals are sent to exciters with common mechanical degrees-of-freedom. A detailed discussion of signal transformation is given in paragraph 6.1, references g and n.

3. CONTROL IMPLEMENTATION.

It is not the intent of this document to provide the specifics of the control algorithms used in the conduct of MESA and MEMA vibration testing. In fact, the various MET control system vendors do not always approach control in the same manner. There are, however, a few basic concepts that are keys to the MESA and MEMA control problem that will be addressed in the following sections.

The theory relative to linear accelerometer placement discussed in Annex A was developed from a time domain perspective. While the time domain approach is very useful in developing an understanding of the basic rigid body kinematics leading to establishing requirements for mapping of acceleration to an arbitrary point (i.e., a drive point), it is not practical to implement as a real time control scheme. In practice, the drive files are generated based on frequency-domain transfer function approximations.

Control system vendors have developed various control algorithms for conduct of a MDOF MET. Although vendors may consider the details of many of their vendor specific techniques to be proprietary, the following general discussion regarding type H_1 transfer function estimations for a MDOF case is still relevant, and serves as a working introduction to the basic control scheme. Basic definitions are reviewed to illustrate the importance of cross- spectrum components in the conduct of a MDOF MET. This discussion is summarized in this Annex and discussed in detail by Bendat and Piersol in paragraph 6.1, reference d.

3.1 SISO Auto and Cross-Spectral Definitions Review.

Prior to matrix-based discussions of transfer function estimates for a MET, consider the following basic scalar definitions as presented by Bendat and Piersol in paragraph 6.1, reference d. The discussions assume two stationary (ergodic) Gaussian random processes $\{x(t)\}$ and $\{y(t)\}$. The finite Fourier Transforms of $\{x(t)\}$ and $\{y(t)\}$ are defined as:

$$X(f) = X(f,T) = \int_0^T x(t) e^{-j2\pi ft}\,dt$$
$$Y(f) = Y(f,T) = \int_0^T y(t) e^{-j2\pi ft}\,dt$$

The auto and cross-spectral densities of $x(t)$ and $y(t)$ for an "unlimited time" length T are defined respectively as:

$$G_{xx}(f) = 2\lim_{T\to\infty}\frac{1}{T}E\left[\left|X(f,T)\right|^2\right]$$
$$G_{yy}(f) = 2\lim_{T\to\infty}\frac{1}{T}E\left[\left|Y(f,T)\right|^2\right]$$
$$G_{xy}(f) = 2\lim_{T\to\infty}\frac{1}{T}E\left[X^*(f)Y(f)\right]$$

Estimates of $G_{xx}(f)$, $G_{yy}(f)$ and $G_{xy}(f)$ as computed over a "finite time" interval are defined as:

$$\tilde{G}_{xx}(f) = S_{xx}(f) = \frac{2}{T}\left[\left|X(f,T)\right|^2\right]$$
$$\tilde{G}_{yy}(f) = S_{yy}(f) = \frac{2}{T}\left[\left|Y(f,T)\right|^2\right]$$
$$\tilde{G}_{xy}(f) = S_{xy}(f) = \frac{2}{T}\left[X^*(f)Y(f)\right]$$

and will have a discrete spectral resolution of $B_e \approx \Delta f = \frac{1}{T}$. Employment of $S_{xx}(f)$, $S_{yy}(f)$ and $S_{xy}(f)$ will generally be unacceptable due to the large random error associated with the "raw" estimate. In practice, the random error is reduced, (refer to paragraph 6.1, reference d, for a detailed error discussion), by computing an ensemble of n_d different averages of length T to obtain a "smooth" estimate defined as:

$$\hat{G}_{xx}(f) = \frac{2}{n_d T}\sum_{i=1}^{n_d}\left[\left|X(f,T)\right|^2\right]$$
$$\hat{G}_{yy}(f) = \frac{2}{n_d T}\sum_{i=1}^{n_d}\left[\left|Y(f,T)\right|^2\right]$$
$$\hat{G}_{xy}(f) = \frac{2}{n_d T}\sum_{i=1}^{n_d}\left[X^*(f)Y(f)\right]$$

3.2 SISO Transfer Function and Coherence Function Definitions Review.

Another very useful tool in the analysis of SISO linear systems are the transfer function and associated coherence estimates. Again, both concepts are explained in detail within paragraph 6.1, reference d. Using the previously defined auto and cross-spectrum definitions, the optimum frequency response function (transfer function) is defined as:

$$\hat{H}_{xy}(f) = \frac{\hat{G}_{xy}(f)}{\hat{G}_{xx}(f)}$$

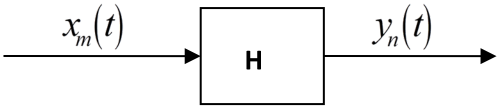
and the associated coherence function is defined as:

$$\hat{\gamma}^2_{xy}(f) = \frac{\left|\hat{G}_{xy}(f)\right|^2}{\hat{G}_{xx}(f)\hat{G}_{yy}(f)}$$

The transfer function provides a frequency domain view of the gain and phase relationship between the input and output signals, while the coherence function indicates the amount of causality in the transfer function. The coherence function range is $0 \le \gamma^2_{xy}(f) \le 1$, with 0 representing no causality and 1 representing perfect causality. Observe that for the SISO case, computation of both $\hat{H}(f)$ and $\gamma_{xy}^2(f)$ are simple division operations to be performed at each of the discrete spectral lines. The following paragraph takes a general MIMO view of the SISO scenario just discussed. In the following discussions, all estimates will be considered to be "smoothed" through the use of an appropriate number of measurements and the ^ symbol will be eliminated.

3.3 MIMO Auto-Spectra, Cross-Spectra, and Initial Function Estimates.

Consider the MIMO system described below consisting of *m* inputs and *n* outputs. Note that, for the general case, $m \neq n$. *(A Linear Time-Invariant (LTI) system is assumed).*



3.3.1 Frequency Domain Transfer Function Relationship.

Develop a Frequency Domain transfer function relationship between the input and output. The following discussion is one of multiple approaches. Welch's method, paragraph 6.1 reference o, is generally used to compute a smoothed estimate of the spectral terms in the following discussion.

a. Define **X(f)** as column vector of the *m* input signals and **Y(f)** as a column vector of the *n* output signals.

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_m \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_n \end{bmatrix}$$

b. Define the Transfer Function Matrix between **X(f)** and **Y(f)** as $\mathbf{H_{xy}(f)}$ such that the *input* precedes the *output*.

$$\mathbf{H_{xy}} = \begin{bmatrix} H_{11} & H_{12} & . & . & H_{1n} \\ H_{21} & H_{22} & . & . & H_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ H_{m1} & H_{m2} & & & H_{mn} \end{bmatrix}$$

c. Define the Instantaneous Power Spectra as:

$\mathbf{S_{xx} = X^{*}X'}$	Instantaneous Input Auto-Spectrum (Dim: *m x m)*
$\mathbf{S_{yy} = Y^{*}Y'}$	Instantaneous Output Auto-Spectrum (Dim: *n x n)*
$\mathbf{S_{xy} = X^{*}Y'}$	Instantaneous Cross-Spectrum (Dim: *m x n)*

d. Define the Cumulative Power Spectra over *k* averages as:

$\mathbf{G_{xx} = \frac{1}{k}\sum_{i=1}^{k} S_{xx\,i}}$	Cumulative Input Auto-Spectrum (Dim: *m x m*)
$\mathbf{G_{yy} = \frac{1}{k}\sum_{i=1}^{k} S_{yy\,i}}$	Cumulative Output Auto-Spectrum (Dim: *n x n*)
$\mathbf{G_{xy} = \frac{1}{k}\sum_{i=1}^{k} S_{xy\,i}}$	Cumulative Cross-Spectrum (Dim: *m x n*)

3.3.2 Key Transfer Function Derivations.

Given the definitions a. and b. above, it follows that:

$$\underset{nx1}{\mathbf{Y}} = \underset{nxm}{\mathbf{H}'_{\mathbf{xy}}} \underset{mx1}{\mathbf{X}}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_n \end{bmatrix} = \begin{bmatrix} H_{11} & H_{21} & . & . & H_{m1} \\ H_{12} & H_{22} & . & . & H_{m2} \\ . & . & . & . & . \\ . & . & . & . & . \\ H_{1n} & H_{2n} & . & . & H_{mn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_m \end{bmatrix}$$

Re-write the input/output relationship in terms of the cumulative auto and cross spectra as defined above in paragraph 3.3.1d.

$$\mathbf{Y}' = \left(\mathbf{H}'_{\mathbf{xy}}\mathbf{X}\right)' = \mathbf{X}'\mathbf{H}_{\mathbf{xy}}$$

$$\mathbf{X}^*\mathbf{Y}' = \mathbf{X}^*\mathbf{X}'\mathbf{H}_{\mathbf{xy}}$$

$$\mathbf{G}_{\mathbf{xy}} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{\mathbf{i=1}}^{\mathbf{k}} \mathbf{X}_{\mathbf{i}}^*\mathbf{Y}_{\mathbf{i}}' = \frac{\mathbf{1}}{\mathbf{k}}\sum_{\mathbf{i=1}}^{\mathbf{k}} \mathbf{X}_{\mathbf{i}}^*\mathbf{X}_{\mathbf{i}}'\mathbf{H}_{\mathbf{xy}} = \left[\frac{\mathbf{1}}{\mathbf{k}}\sum_{\mathbf{i=1}}^{\mathbf{k}} \mathbf{X}_{\mathbf{i}}^*\mathbf{X}_{\mathbf{i}}'\right]\mathbf{H}_{\mathbf{xy}} = \mathbf{G}_{\mathbf{xx}}\mathbf{H}_{\mathbf{xy}}$$

$$\mathbf{G}_{\mathbf{xy}} = \mathbf{G}_{\mathbf{xx}}\mathbf{H}_{\mathbf{xy}}$$

$$\mathbf{G}_{\mathbf{xx}}^{-1}\mathbf{G}_{\mathbf{xy}} = \mathbf{G}_{\mathbf{xx}}^{-1}\mathbf{G}_{\mathbf{xx}}\mathbf{H}_{\mathbf{xy}}$$

$$\underset{\mathbf{mxm}}{\mathbf{G}_{\mathbf{xx}}^{-1}}\ \underset{\mathbf{mxn}}{\mathbf{G}_{\mathbf{xy}}} = \underset{\mathbf{mxn}}{\mathbf{H}_{\mathbf{xy}}}$$

In performing laboratory MET, the initial estimation of $\mathbf{H}_{\mathbf{xy}}$ will be computed based on a set of *uncorrelated random input signals.* The desired signal, $\mathbf{Y}$, will have been either measured directly, or possibly computed via a 6-DOF model based prediction, leaving $\mathbf{X}$ (that will represent the input to the vibration exciter) as the unknown.

Recall that $\underset{\mathbf{nx1}}{\mathbf{Y}} = \underset{\mathbf{nxm}}{\mathbf{H}'_{\mathbf{xy}}}\ \underset{\mathbf{mx1}}{\mathbf{X}}$, therefore, $\left(\mathbf{H}'_{\mathbf{xy}}\right)^{-1}\mathbf{Y} = \left(\mathbf{H}'_{\mathbf{xy}}\right)^{-1}\mathbf{H}'_{\mathbf{xy}}\mathbf{X}$ yielding $\underset{\mathbf{mxn}}{\left(\mathbf{H}'_{\mathbf{xy}}\right)^{-1}}\ \underset{\mathbf{nx1}}{\mathbf{Y}} = \underset{\mathbf{mx1}}{\mathbf{X}}$.

Note that for the general case in which *m≠n,* the computation of $\left(\mathbf{H}'_{\mathbf{xy}}\right)^{-1}$ will require a pseudo-inverse (Moore-Penrose) approximation. This computation involves a singular value decomposition (SVD) of $\mathbf{H}'_{\mathbf{xy}}$. Viewing the singular values provides two useful pieces of information. First, it provides information on a spectral line basis as to the rank of $\mathbf{H}'_{\mathbf{xy}}$, and second, it provides an indication as to the dynamic range of $\mathbf{H}'_{\mathbf{xy}}$, thereby providing insight into the potential for noise in computation of the drive files. Estimations of $\mathbf{H}'_{\mathbf{xy}}$ via SVD techniques are more computationally intense than classical methods such as the Cholesky decomposition; however, the SVD technique is more robust and capable of addressing rectangular and singular matrices. SVD techniques also provide straight forward methods of addressing dynamic range and noise by investigating the ratio of the largest to smallest singular values.

From a Procedure II control algorithm perspective, one may be interested in computation of $\mathbf{G}_{\mathbf{xx}}$ directly from $\mathbf{H}_{\mathbf{xy}}$. Recall from above that $\underset{\mathbf{nx1}}{\mathbf{Y}} = \underset{\mathbf{nxm}}{\mathbf{H}'_{\mathbf{xy}}}\ \underset{\mathbf{mx1}}{\mathbf{X}}$, from which the following is derived:

$$\mathbf{Y} = \mathbf{H}_{xy}^{'}\mathbf{X}$$

$$\mathbf{Y}^{'} = \left(\mathbf{H}_{xy}^{'}\mathbf{X}\right)^{'} = \mathbf{X}^{'}\mathbf{H}_{xy}$$

$$\mathbf{Y}^{*} = \left(\mathbf{H}_{xy}^{'}\mathbf{X}\right)^{*} = \mathbf{H}_{xy}^{'*}\mathbf{X}^{*}$$

$$\mathbf{Y}^{*}\mathbf{Y}^{'} = \left(\mathbf{H}_{xy}^{'*}\mathbf{X}^{*}\right)\left(\mathbf{X}^{'}\mathbf{H}_{xy}\right)$$

This yields:

$$\mathbf{G}_{yy} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{Y}_i^{*}\mathbf{Y}_i^{'} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{H}_{xy}^{*'}\mathbf{X}_i^{*}[\mathbf{X}_i^{'}\mathbf{H}_{xy}] = \mathbf{H}_{xy}^{*'}\left[\frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{X}_i^{*}\mathbf{X}_i^{'}\right]\mathbf{H}_{xy} = \mathbf{H}_{xy}^{*'}\mathbf{G}_{xx}\mathbf{H}_{xy}$$

$$\mathbf{G}_{yy} = \mathbf{H}_{xy}^{'*}\mathbf{G}_{xx}\mathbf{H}_{xy}$$

Which leads directly to:

$$\mathbf{G}_{xx} = \left(\mathbf{H}_{xy}^{'*}\right)^{-1}\mathbf{G}_{yy}\left(\mathbf{H}_{xy}\right)^{-1}$$

Paragraph 6.1, reference d, goes into considerably more detail, to include error analysis, regarding the discussion above. In addition, the various control system vendors continue to improve on the basic concepts using unique (and often proprietary) techniques to improve convergence to the reference array based on error in both time and frequency domains. The discussion above serves as an illustration through use of well defined and established analyses of the increased level of complexity associated with MDOF vibration testing. Of particular interest are that the fundamental principles are based on the assumption that the excitation system is LTI, and that the reference measurements were acquired from a kinematically consistent body. Clearly, neither assumption holds for the majority of laboratory vibration tests, even in the SESA case. The issue at hand is establishing metrics of acceptability for a MET.

3.3.3 Key Transfer Function Derivations Alternative.

An alternative to the derivations in paragraphs 3.3.1 and 3.3.2, which is commonly employed in the MIMO vibration control arena, is based on making the following minor changes in definitions within paragraph 3.3.1:

a. Define **X(f)** as column vector of the *m* input signals and **Y(f)** as a column vector of the *n* output signals as defined in paragraph 3.3.1.

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_m \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_n \end{bmatrix}$$

b. Define the Transfer Function Matrix between **X(f)** and **Y(f)** as **H_{yx}(f)** such that the *output* precedes the *input*. Recalling $\mathbf{H}_{xy}$ as defined in paragraph 3.3.1, observe that $\mathbf{H}_{yx} = \mathbf{H}_{xy}^{'}$ and that $\mathbf{H}_{xy}^{-1} \neq \mathbf{H}_{yx}$.

$$\mathbf{H}_{yx} = \begin{bmatrix} H_{11} & H_{21} & . & . & H_{m1} \\ H_{12} & H_{22} & . & . & H_{m2} \\ . & . & . & . & . \\ . & . & . & . & . \\ H_{1n} & H_{2n} & . & . & H_{mn} \end{bmatrix}$$

$$\underset{\mathbf{nx1}}{\mathbf{Y}} = \underset{\mathbf{nxm}}{\mathbf{H}_{yx}}\ \underset{\mathbf{mx1}}{\mathbf{X}}$$

c. Define the Instantaneous Power Spectra as:

$\hat{\mathbf{\Phi}}_{xx} = \mathbf{XX}^{*'}$ Instantaneous Input Auto-Spectrum (Dim: *m x m)*

$\hat{\mathbf{\Phi}}_{yy} = \mathbf{YY}^{*'}$ Instantaneous Output Auto-Spectrum (Dim: *n x n)*

$\hat{\mathbf{\Phi}}_{yx} = \mathbf{YX}^{*'}$ Instantaneous Cross-Spectrum (Dim: *n x m)*

Observe in comparison to the definitions provided in paragraph 3.3.1 that:

$$\hat{\mathbf{\Phi}}_{xx} = \mathbf{S}_{xx}', \quad \hat{\mathbf{\Phi}}_{yy} = \mathbf{S}_{yy}', \quad \textbf{and} \ \hat{\mathbf{\Phi}}_{yx} = \mathbf{S}_{xy}^{*'}$$

d. Define the Cumulative Power Spectra over *k* averages as:

$\mathbf{\Phi}_{xx} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\hat{\mathbf{\Phi}}_{xx\,i}$ Cumulative Input Auto-Spectrum (Dim: *m x m*)

$\mathbf{\Phi}_{yy} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\hat{\mathbf{\Phi}}_{yy\,i}$ Cumulative Output Auto-Spectrum (Dim: *n x n*)

$\mathbf{\Phi}_{yx} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\hat{\mathbf{\Phi}}_{yx\,i}$ Cumulative Cross-Spectrum (Dim: *n x m*)

Observe in comparison to the definitions provided in paragraph 3.3.1 that:

$$\mathbf{\Phi}_{xx} = \mathbf{G}_{xx}', \quad \mathbf{\Phi}_{yy} = \mathbf{G}_{yy}', \quad \textbf{and} \ \mathbf{\Phi}_{yx} = \mathbf{G}_{xy}^{*'}$$

Applying the input/output relationship of an LTI system, and by making the following substitutions based on the definitions for the cumulative auto and cross spectra as defined above in paragraphs 3.3.3c and 3.3.3d yields the following:

$$\mathbf{\Phi}_{yy} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{Y}_i\mathbf{Y}_i^{*'} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{H}_{yx}\mathbf{X}_i[\mathbf{H}_{yx}\mathbf{X}_i]^{*'} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{H}_{yx}\mathbf{X}_i\mathbf{X}_i^{*'}\mathbf{H}_{yx}^{*'} = \mathbf{H}_{yx}\mathbf{\Phi}_{xx}\mathbf{H}_{yx}^{*'} \quad \text{and,}$$

$$\mathbf{\Phi}_{xx} = \mathbf{H}_{yx}^{-1}\mathbf{\Phi}_{yy}\left[\mathbf{H}_{yx}^{*'}\right]^{-1} \quad \text{Or, by defining } \mathbf{Z} = \mathbf{H}_{yx}^{-1} \text{ simplifies to } \mathbf{\Phi}_{xx} = \mathbf{Z\Phi}_{yy}\mathbf{Z}^{*'}$$

$$\mathbf{\Phi}_{yx} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{Y}_i\mathbf{X}_i^{*'} = \frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{H}_{yx}\mathbf{X}_i\mathbf{X}_i^{*'} = \mathbf{H}_{yx}\frac{\mathbf{1}}{\mathbf{k}}\sum_{i=1}^{k}\mathbf{X}_i\mathbf{X}_i^{*'} = \mathbf{H}_{yx}\mathbf{\Phi}_{xx} \quad \textit{which leads to}:$$

$$\mathbf{\Phi}_{yx}\mathbf{\Phi}_{xx}^{-1} = \mathbf{H}_{yx}$$

Observe that two approaches discussed within paragraph 3.3 are very similar in structure. Selection of technique is generally one of preference or possibly computational advantage.

3.4 MIMO Coherence Functions.

The concept of coherence will need to be expanded to address the MIMO case. Refer to the paragraph 6.1, references d and l, for a detailed discussion on this subject. Following, are three basic coherence definitions that apply to the MIMO case for a linear system.

3.4.1 Ordinary Coherence.

The ordinary coherence function is defined as the correlation coefficient describing the linear relationship between any two single spectra. In the multiple input case, care must be taken in interpretation of ordinary coherence. It is possible that the coherence between the output and a given input may be much less than unity, even if the relationship is strictly linear due to the influence of other input signals. For a linear MIMO system, the ordinary coherence is defined as:

$$\gamma^2_{mn}(f) = \frac{\left|G_{xy_{mn}}\right|^2}{G_{xx_{mm}}G_{yy_{nn}}} \quad \text{where,}$$

$G_{xx_{mm}}(f)$ = auto-spectrum of the input m

$G_{yy_{nn}}(f)$ = auto-spectrum of the output n

$G_{xy_{mn}}(f)$ =cross-spectrum between input m and output n

3.4.2 Partial Coherence.

The partial coherence function is defined as the ordinary coherence between one conditioned output and another conditioned output, between one conditioned input and another conditioned input, or between one conditioned input and a conditioned output. The individual input and output signals are "conditioned" by removing the contributions from other inputs. There is a partial coherence function that exists for every input-input, output-output, and input-output combination for all permutations of conditioning.

3.4.3 Multiple Coherence.

The multiple coherence function is defined as the correlation coefficient describing the linear relationship between a given output and all known inputs. A multiple coherence function exists for each output signal. The multiple coherence function provides an excellent method of evaluating the degree and relative importance of unknown contributions such as noise and nonlinearities to each output signal.

As is the case for ordinary coherence, a low multiple coherence value represents a low causality between the output signal of interest and the input signals. This information is critical in the closed loop control process in that it will influence the transfer function estimate. In fact, MDOF control systems use the multiple coherence function as a key test parameter. Specifically, the control algorithm will compute the multiple coherence for each output channel at each spectral line. Prior to updating the transfer function during a test, the multiple coherence function will be evaluated to ensure a specific threshold is achieved, (i.e. $\gamma_{mn}^2(f) \geq 0.7$). If the user-defined threshold has not been achieved, the transfer function for that spectral line will not be updated. Partial and multiple coherence are discussed in detail in paragraph 6.1, reference d. Underwood also provides an interesting perspective of both partial and multiple coherence in paragraph 6.1, reference l.

3.5 Drive Signal Compensation.

The previous discussions of auto and cross-spectral densities and how they are used in the computation of the system transfer function and associated coherence functions are all applied in the initial system identification phase in a MET. Subsequent to the initial system identification, the output (drive) signals are updated similar to the traditional SESA case. Although the details of each control system vendor's algorithms will vary, there are two basic drive signal update methodologies.

The first drive signal update technique is based simply on continuous updates of the system transfer function, and is performed throughout the duration of the test to address minor system changes (paragraph 6.1, reference m). Note that for any frequencies for which the drive signals are fully correlated, corrections to the system transfer function will not be possible.

The second drive signal update technique is based on the error spectrum that is computed between the feedback spectrum and the specified reference spectrum. Typically, some fraction of the error is applied to a correction of the coupling matrix corrected during each loop. The coupling matrix is the spectral density matrix that couples the vector of white noise sources generated by the control system to achieve the desired reference spectrum.

METHOD 527.2, ANNEX C

PROCEDURE I MET (TIME WAVEFORM REPLICATION (TWR) SPECIFIC)

1. PROCEDURE I MET (TIME DOMAIN REFERENCE CRITERIA).

1.1 Preprocessing.

Since placement and orientation of transducers are paramount in the conduct of MDOF MET, performing a thorough pretest review is essential to overall test validity and efficiency. Misalignment of one transducer will adversely affect the transfer function matrix as a whole. To address these types of issues, take detailed measurements and photographs of the actual field setup (i.e., how and where the materiel was mounted) to aid in proper laboratory setup (since the laboratory configuration should mimic the field setup as accurately as possible). In addition, once the test item and associated measurement and control instrumentation are configured in the laboratory, examine phase and coherence measurements between drive channels and control channels to make sure that input points and their resultant responses are logical (e.g., a vertical input should largely affect vertical responses at low frequencies). Also, ensure the spectral characteristics of the control accelerometers and associated signal conditioning equipment have the same frequency response characteristics as that of the instrumentation used to make the original reference measurements, or properly pre-condition data as required to ensure proper phase relationships between channels.

2. ANALYSIS CONSIDERATIONS FOR A PROCEDURE I MET.

2.1 Addressing Translational Motion.

Since linear transducers are generally the measurement transducers of choice, translational measurements will be readily available. One needs only to have a well-defined coordinate system established.

2.2 Addressing Angular Motion.

Auto-Spectral Density (ASD) analysis provides a general spectral view of the reference data; however, it contains no phase information. It is the differences in phase and amplitude between collinear accelerometers that indicate the presence of angular motion. One method of investigating the presence of angular acceleration (either pure or combined with translational acceleration) from a suite of linear accelerometers is to perform complex transfer functions between collinear pairs of linear accelerometers. Subsequently, performing the same transfer function analysis between the same locations in the laboratory provides another metric for measuring the fidelity of the laboratory test. Analyzing the transfer functions corresponding to the field and laboratory measurements often indicates where the mechanical impedance between field and laboratory begin to diverge. Referring back to the ASD measurements, one is able to gain some perspective as to the amount of energy present as a function of frequency, providing perspective into the deviations expected as a result of divergence in mechanical impedance. Similarities between the reference and laboratory transfer functions indicate field and laboratory rotations are also similar.

In an effort to address the actual level and fidelity associated with rotational degrees-of-freedom from a test controlled entirely by feedback obtained from linear accelerometers, computations of angular motion can be developed. Perform computations from both the reference data and corresponding laboratory control accelerometer pairs, and compare results. The computation takes the form of a small angle approximation; however, since the reference plane on which the accelerometer is mounted is actually rotating, there is no computation error as a function of angle as in the case of a fixed plane small angle approximation. To illustrate, consider two linear accelerometers positioned to measure z-axis motion mounted a distance l inches from their relative centerline along the y-axis. An estimate of Roll (Rx) axis angular motion in units of $\frac{rad}{s^2}$ at the centerline between the two transducers can be computed as $\frac{(a_{1z} - a_{2z})*386}{2l}$.

Ideally this technique will provide a good metric for analyzing the angular motion for the "rigid body" case. The frequency, at which the field data and laboratory data begin to diverge is an indication of where the mechanical impedance between tactical field mechanical interface and laboratory fixturing begins to differ. The magnitude of the divergence provides some idea of the quality of the impedance match, and provides a key data point in understanding if the test fidelity is sufficient in addressing a test-specific criteria. In general, the instantaneous center of rotation (ICR) may not coincide exactly with the ICR of the test platform, and that the angular motion estimates may, in fact, be vectors that are not perfectly orthogonal with respect to the true axis of rotation. However, as long as the laboratory reference linear accelerometers used to make the angular acceleration estimates correlate to the exact location and phase of the reference measurements, a localized angular motion comparison is still of interest in addressing replication fidelity.

If possible, even though it may be band-limited, recommend an angular accelerometer or rate transducer be placed at the midpoint between the linear accelerometers being used to estimate the rotational DOF of interest. The addition of the angular accelerometer will provide a direct measure of ground truth for angular acceleration at a particular point on a structure.

3. TEST TOLERANCES FOR A PROCEDURE I MET.

As discussed in paragraph 4.2.2, at this point in TWR test philosophy, test tolerance specification is not well quantified. However, numerous candidates for quantifying TWR testing are provided in the Annex section of Method 525.2. Each of the metrics addressed in Method 525.2-Annex A for SESA TWR is also applicable to the MDOF case, only the MDOF case will consist of an “array” of reference channels and an “array” of control channels. As is the case for SESA TWR, recommend the reference time histories be segmented into categories of stationary random, shock, or non-stationary, and the tolerance criteria be applied to each segment based on the data classification. For tolerance development purposes for TWR, the tolerances ***should not exceed*** the tolerances provided in Methods 514.8, 516.8, and 519.8 respectively, for stationary random vibration and mechanical shock categories. The tolerances for the third form of time trace, non-stationary data, are somewhat dependent on the nature of the non-stationarity. Techniques for non-stationarity assessment for which time trace amplitude is a function of both time and frequency are available (see paragraph 6.1, reference d). Some non-stationary time traces that have time invariant frequency characteristics can be represented by the Product Model (PM), and can be processed for tolerance purposes as stationary random vibration with a time-varying envelope. Consult Annexes A and B of Method 525.2 for details of TWR tolerance specification for non-stationary time traces. Finally, in addition to time segmenting the overall reference and control traces, it may be desirable to establish separate test tolerances over common bandwidths of the reference and control time traces, i.e., perform frequency segmenting. This could be accomplished through digital filter scheme. This Method provides no guidance for tolerance development under frequency segmentation.

3.1 Composite (Global) Error Discussion for Procedure I.

One obvious point of concern in addressing adequacy of a 6-DOF TWR test is in a global sense. This is analogous, in the conduct of traditional SDOF testing to the practice of providing a composite control plot summarizing multiple control channel averaging or weighting schemes. For example, experience has shown that in MEMA tests in which a specific mechanical degree-of-freedom consists of a very small percentage of the composite energy across all mechanical degrees-of-freedom, the associated error for that DOF will often be higher than the desired test tolerances discussed in paragraph 3 above. Three candidates, (many others are possible) for accessing global error are addressed in paragraph 6.1, reference k, and summarized below. The three techniques discussed below are consistent with the rudimentary division of data types discussed in Method 525.2, Annex A.

3.2 Global RMS Error.

One of the most common time domain error metrics employed in TWR testing is simply comparisons between the reference data and laboratory data as EU-rms versus time computed over short time slices for the duration of the test. For the MDOF TWR case, the rms versus time error is easily calculated for each control channel as illustrated by Step 2 below. Also of interest would be an energy weighted view of the rms versus time error between the reference and control signals. This concept is developed in the following steps:

Step 1 The arrays $\underset{JxN}{r}$ and $\underset{JxN}{l}$ shown in Equation 3.2.1 represent, respectively, the N point sampled reference and laboratory test data for each of the J control channels. Test-specific parameters such as sample frequency and filter settings should be tracked by the test operator. It is assumed that the time histories represented by Equation 3.2.1 will not have a bias, or that any bias has been removed during pre-processing.

$$\underset{JxN}{r(n)} = \begin{pmatrix} r_1(n) \\ r_2(n) \\ \vdots \\ r_J(n) \end{pmatrix} \quad \underset{JxN}{l(n)} = \begin{pmatrix} l_1(n) \\ l_2(n) \\ \vdots \\ l_J(n) \end{pmatrix} \quad n = 1, 2 \ldots N; \qquad (3.2.1)$$

Step 2 The two matrices RMS_r and RMS_l shown in Equation 3.2.2 contain the g-rms values for each reference and laboratory test channel computed over each time segment, s. The j index, $j = 1, \cdots J$

, represents the control channel number and the s index, $s = 1,\cdots S$, represents the time segment number. For example, if the sample frequency Fs is 1024 Hz, and the rms calculation is to be computed every 0.5 seconds (M=512 samples), $s=1$ would represent samples $n = 1\cdots M$, $s=2$ would represent the samples $n = M+1\cdots 2M$, and so on.

$$\underset{JxS}{RMS\_r} = \begin{pmatrix} rms\_r_{11} & \dots & rms\_r_{1S} \\ \vdots & \ddots & \vdots \\ rms\_r_{J1} & \cdots & rms\_r_{JS} \end{pmatrix} \quad \underset{JxS}{RMS\_l} = \begin{pmatrix} rms\_l_{11} & \dots & rms\_l_{1S} \\ \vdots & \ddots & \vdots \\ rms\_l_{J1} & \cdots & rms\_l_{JS} \end{pmatrix} \tag{3.2.2}$$

$$where, \quad rms\_r_{js} = \sqrt{\frac{1}{M}\sum_{n=((M.s)-M+1}^{M.s} r_j^{\,2}(n)} \quad and \quad rms\_l_{js} = \sqrt{\frac{1}{M}\sum_{n=((M.s)-M+1}^{M.s} l_j^{\,2}(n)}$$

Step 3 Observing that the columns of the two matrices shown in Equation 3.2.2 represent the reference and laboratory test channels, g-rms values for a given time segment s, it is possible to isolate the individual columns and develop a weighting strategy across all control channels for each time segment. Equation 3.2.3 illustrates a 2-norm computed for each column of the reference matrix RMS_r. Note that post multiplication by indexing vector U_s provides a method of isolating the s^{th} column of interest.

$$\underset{1xS}{nc\_rms\_r} = \left(\left\| \left(RMS\_r\right)U_1 \right\|_2, \left\| \left(RMS\_r\right)U_2 \right\|_2, \cdots, \left\| \left(RMS\_r\right)U_S \right\|_2 \right)$$

$$where, \quad \underset{Sx1}{U_1} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \underset{Sx1}{U_2} = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad \cdots \quad ,\underset{Sx1}{U_S} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \tag{3.2.3}$$

Step 4 Equation 3.2.4 demonstrates computation of a weighting factor for each entry in the reference matrix RMS_r, based on a column normalization to the corresponding 2-norm computed in Equation 3.2.3. This weighting factor may be considered in addressing rms-error between the reference and laboratory data.

$$\underset{JxS}{Wt} = \begin{pmatrix} \frac{\left(RMS\_r_{11}\right)^2}{\left(nc\_rms\_r_1\right)^2} & \frac{\left(RMS\_r_{12}\right)^2}{\left(nc\_rms\_r_2\right)^2} & \cdots & \frac{\left(RMS\_r_{1S}\right)^2}{\left(nc\_rms\_r_S\right)^2} \\ \frac{\left(RMS\_r_{21}\right)^2}{\left(nc\_rms\_r_1\right)^2} & \frac{\left(RMS\_r_{22}\right)^2}{\left(nc\_rms\_r_2\right)^2} & \cdots & \frac{\left(RMS\_r_{2S}\right)^2}{\left(nc\_rms\_r_S\right)^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\left(RMS\_r_{J1}\right)^2}{\left(nc\_rms\_r_1\right)^2} & \frac{\left(RMS\_r_{J2}\right)^2}{\left(nc\_rms\_r_2\right)^2} & \cdots & \frac{\left(RMS\_r_{JS}\right)^2}{\left(nc\_rms\_r_S\right)^2} \end{pmatrix} \tag{3.2.4}$$

Step 5 The relative error between the reference signals and signals measured during laboratory testing can be computed on a log scale per Equation 3.2.5.

$$\underset{JxS}{RMS\_err} = 20\log_{10}\begin{pmatrix} \left(\dfrac{RMS\_l_{11}}{RMS\_r_{11}}\right) & \left(\dfrac{RMS\_l_{12}}{RMS\_r_{12}}\right) & \cdots & \left(\dfrac{RMS\_l_{1S}}{RMS\_r_{1S}}\right) \\ \left(\dfrac{RMS\_l_{21}}{RMS\_r_{21}}\right) & \left(\dfrac{RMS\_l_{22}}{RMS\_r_{22}}\right) & \cdots & \left(\dfrac{RMS\_l_{2S}}{RMS\_r_{2S}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\dfrac{RMS\_l_{J1}}{RMS\_r_{J1}}\right) & \left(\dfrac{RMS\_l_{J2}}{RMS\_r_{J2}}\right) & \cdots & \left(\dfrac{RMS\_l_{JS}}{RMS\_r_{JS}}\right) \end{pmatrix} \tag{3.2.5}$$

Step 6 The RMS_err matrix can be normalized by the weighting parameter defined in Matrix Wt as illustrated in Equation 3.2.6.

$$\underset{JxS}{RMS\_Nerr} = \begin{pmatrix} (RMS\_err_{11})(Wt_{11}) & (RMS\_err_{12})(Wt_{12}) & \cdots & (RMS\_err_{1S})(Wt_{1S}) \\ (RMS\_err_{21})(Wt_{21}) & (RMS\_err_{22})(Wt_{22}) & \cdots & (RMS\_err_{2S})(Wt_{2S}) \\ \vdots & \vdots & \ddots & \vdots \\ (RMS\_err_{J1})(Wt_{J1}) & (RMS\_err_{J2})(Wt_{J2}) & \cdots & (RMS\_err_{JS})(Wt_{JS}) \end{pmatrix} \tag{3.2.6}$$

Step 7 A Global-rms error may now be established for each time segment as illustrated in Equation 3.2.7.

$$\underset{1xS}{Glob\_rms\_err} = \left(\sum_{j=1}^{J}(RMS\_Nerr)U_1,\cdots,\sum_{j=1}^{J}(RMS\_Nerr)U_S\right) \tag{3.2.7}$$

The rms error produced in Step 7 above provides a global perspective to rms error between the reference and laboratory data in which each control location is included and weighted in terms of the energy within each time segment, *s*.

3.3 Global ASD Error.

One of the most common frequency domain error metrics employed in TWR testing is based upon comparisons of $ASD's$ computed over a given time segment. The level of non-stationarity of a reference signal and/or similarities in the data over a particular segment of time may be considered in selection of the time segment over which the ASD is computed. While it is certainly easy to argue the usefulness of an ASD estimate of non-stationary data, the technique is still useful in making a direct comparison between field based reference signals and laboratory-based data from a TWR test. A logical division of time segments is to select the segments to be as close to piecewise stationary as possible.

As previously stated, the topic of this document is centered on establishing global performance metrics for the MDOF TWR scenario. The steps that follow outline one technique for consideration in viewing ASD results computed over multiple control channels.

Step 1 The arrays $\underset{JxN}{r}$ and $\underset{JxN}{l}$ shown in Equation 3.3.1 represent respectively, the N point sampled reference and laboratory data for each of the J control channels. Test-specific parameters such as sample frequency, Fs, and filter settings, should be tracked by the test operator. It is assumed that the time histories represented by Equation 3.3.1 will not have a bias, or that any bias has been removed during pre-processing.

$$\underset{JxN}{r(n)} = \begin{pmatrix} r_1(n) \\ r_2(n) \\ \vdots \\ r_J(n) \end{pmatrix} \quad \underset{JxN}{l(n)} = \begin{pmatrix} l_1(n) \\ l_2(n) \\ \vdots \\ l_J(n) \end{pmatrix} \quad n = 1,2\ldots N; \tag{3.3.1}$$

Step 2 The two matrices ASD_r_s and ASD_l_s shown in Equation 3.3.2 represent ASD estimates computed over time segment, s. The j index, $j = 1,\cdots J$, represents the control channel number and the f index, $f = 1,\cdots F$, $where \quad F = \frac{BS}{2}$, represents each spectral line of the ASD estimate. For example, if $Fs = 1024$ and the block-size (BS) used in the estimate of the ASD is set to $BS = 512$, $F = 256$ and the frequency resolution would be $\Delta f = \frac{Fs}{BS} = 2Hz$. In computing the ASD estimates, the time segment, s, may be either the entire range $n = 1...N$, or some subset thereof.

$$\underset{JxF}{ASD\_r_s}(f) = \begin{pmatrix} asd\_r_{11} & \cdots & asd\_r_{1F} \\ \vdots & \ddots & \vdots \\ asd\_r_{J1} & \cdots & asd\_r_{JF} \end{pmatrix} \quad \underset{JxF}{ASD\_l_s} = \begin{pmatrix} asd\_l_{11} & \cdots & asd\_l_{1F} \\ \vdots & \ddots & \vdots \\ asd\_l_{J1} & \cdots & asd\_l_{JF} \end{pmatrix} \tag{3.3.2}$$

Step 3 Observing that the columns of the two matrices shown in Equation 3.3.2 represent the reference and laboratory test channels $\frac{G^2}{Hz}$ values for a given spectral line as estimated over time segment, s, the individual columns can be isolated and a weighting strategy developed across all control channels for each spectral line. Equation 3.3.3 illustrates a 2-norm computed for each column of the reference matrix ASD_r_s. Post multiplication by indexing vector, U, provides a method of isolating an individual column of interest.

$$\underset{1xF}{nc\_asd\_r_s} = \left(\left\| (ASD\_r_s)U_1 \right\|_2, \left\| (ASD\_r_s)U_2 \right\|_2, \cdots, \left\| (ASD\_r_s)U_F \right\|_2 \right)$$

$$where, \quad \underset{Fx1}{U_1} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \underset{Fx1}{U_2} = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad \cdots, \underset{Fx1}{U_F} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \tag{3.3.3}$$

Step 4 Equation 3.3.4 demonstrates computation of a weighting factor for each entry in the reference matrix ASD_r based on a column normalization to the corresponding 2-norm computed in Equation 3.3.3. This weighting factor may be considered in addressing $\frac{G^2}{Hz}$ error between the reference and laboratory data.

$$\underset{JxF}{Wt_s} = \begin{pmatrix} \frac{(ASD\_r_{11})^2}{(nc\_asd\_r_1)^2} & \frac{(ASD\_r_{12})^2}{(nc\_asd\_r_2)^2} & \cdots & \frac{(ASD\_r_{1F})^2}{(nc\_asd\_r_F)^2} \\ \frac{(ASD\_r_{21})^2}{(nc\_asd\_r_1)^2} & \frac{(ASD\_r_{22})^2}{(nc\_asd\_r_2)^2} & \cdots & \frac{(ASD\_r_{2F})^2}{(nc\_asd\_r_F)^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{(ASD\_r_{J1})^2}{(nc\_asd\_r_1)^2} & \frac{(ASD\_r_{J2})^2}{(nc\_asd\_r_2)^2} & \cdots & \frac{(ASD\_r_{JF})^2}{(nc\_asd\_r_F)^2} \end{pmatrix} \tag{3.3.4}$$

Step 5 The relative error between the reference signals and signals measured during laboratory testing can be computed on a log scale per Equation 3.3.5.

$$\underset{JxF}{ASD\_err_s} = 10\log_{10}\begin{pmatrix} \left(\frac{ASD\_l_{11}}{ASD\_r_{11}}\right) & \left(\frac{ASD\_l_{12}}{ASD\_r_{12}}\right) & \cdots & \left(\frac{ASD\_l_{1F}}{ASD\_r_{1F}}\right) \\ \left(\frac{ASD\_l_{21}}{ASD\_r_{21}}\right) & \left(\frac{ASD\_l_{22}}{ASD\_r_{22}}\right) & \cdots & \left(\frac{ASD\_l_{2F}}{ASD\_r_{2F}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{ASD\_l_{J1}}{ASD\_r_{J1}}\right) & \left(\frac{ASD\_l_{J2}}{ASD\_r_{J2}}\right) & \cdots & \left(\frac{ASD\_l_{JF}}{ASD\_r_{JF}}\right) \end{pmatrix} \tag{3.3.5}$$

Step 6 The ASD_err matrix can be normalized by the weighting parameter defined in Matrix Wt as illustrated in Equation 3.3.6.

$$\underset{JxF}{ASD\_Nerr_s} = \begin{pmatrix} (ASD\_err_{11})(Wt_{11}) & (ASD\_err_{12})(Wt_{12}) & \cdots & (ASD\_err_{1F})(Wt_{1F}) \\ (ASD\_err_{21})(Wt_{21}) & (ASD\_err_{22})(Wt_{22}) & \cdots & (ASD\_err_{2F})(Wt_{2F}) \\ \vdots & \vdots & \ddots & \vdots \\ (ASD\_err_{J1})(Wt_{J1}) & (ASD\_err_{J2})(Wt_{J2}) & \cdots & (ASD\_err_{JF})(Wt_{JF}) \end{pmatrix} \tag{3.3.6}$$

Step 7 A Global ASD error may now be established for each time segment, s, as illustrated in Equation 3.3.7.

$$\underset{1xF}{Glob\_asd\_err_s} = \left(\sum_{j=1}^{J} (ASD\_Nerr) U_1, \cdots, \sum_{j=1}^{J} (ASD\_Nerr) U_F \right) \tag{3.37}$$

The ASD error spectrum produced in Step 7 above provides a global perspective to ASD error between the reference and laboratory data in which each control location is included, and weighted in terms of the energy at each spectral line.

3.4 Global SRS Error.

As discussed in Method 525.2, significant transients that can be identified within a reference time trace may be analyzed post-test using traditional SRS or pseudo velocity SRS analysis. A global error technique for SRS analysis can be developed with a slight variation of the ASD approach defined in paragraph 3.3 above. Specifically, as a substitute for indexing on a frequency line basis, index frequency on a 1/12th octave basis using maxi-max acceleration within each 1/12th octave band.

METHOD 527.2, ANNEX D

PROCEDURE II MET (SPECTRAL DENSITY MATRIX (SDM) SPECIFIC)

1. PROCEDURE II MET (FREQUENCY DOMAIN REFERENCE CRITERIA).

1.1 Preprocessing.

Since placement and orientation of transducers are paramount in the conduct of MDOF MET, performing a thorough pretest review is essential to overall test validity and efficiency. Misalignment of one transducer will adversely affect the transfer function matrix as a whole. To address these types of issues, take detailed measurements and photographs of the actual setup (i.e., how and where the item was mounted) to aid in proper laboratory setup (since it should mimic the field setup as accurately as possible). In addition, once the test item and associated measurement and control instrumentation are configured in the laboratory, examine phase and coherence measurements between drive channels and control channels to make sure that input points and their resultant responses are logical (e.g., a vertical input should largely affect vertical responses at low frequencies). Ensure the spectral characteristics of the control accelerometers and associated signal conditioning equipment have the same spectral characteristics of the instrumentation used to make the original reference measurements, or properly pre-condition data as required, to ensure proper phase relationships between channels. Also, it is highly recommended that an FEM model of the MET configuration be developed. A prior knowledge of the modal characteristics of a laboratory-based MET system often proves to be of great value in addressing undesired modal response through implementation of additional feedback to be considered in the control scheme.

2. ANALYSIS CONSIDERATIONS FOR A PROCEDURE II MET.

2.1 MESA and MEMA Specification Parameters.

The classical metrics addressed in Method 514.8 for control of SESA vibration tests are insufficient for the analysis of a MET. In the conduct of either a MESA or MEMA Procedure II vibration test, both auto-spectral density (ASD) and cross-spectral density (CSD) terms are required test parameters. As one would expect, the configuration of a MET will influence the reference spectral requirements. For example, consider defining a random test for the two MET systems illustrated in Figures 527.2-2 and 527.2-3. Table 527.2D-I illustrates a spectral density matrix (SDM) construct, the 2-DOF MET shown in Figure 527.2-2 and similarly, Table 527.2D-II illustrates the format of spectral information required in specifying the 3-DOF MET of the system shown in Figure 527.2-3. Observe that the format of a Spectral Density Matrix (SDM) consists of auto-spectral density (power spectral density) terms on the diagonal and cross-spectral density terms on the off-diagonal. Also, note the Hermitian structure for the case in which the SDM is square.

Table 527.2D-I. Reference criteria for a 2-DOF linear motion random MET.

$ASD_{z1z1}(f)$	$CSD^{*}_{z1z2}(f)$
$CSD_{z1z2}(f)$	$ASD_{z2z2}(f)$

Table 527.2D-II. Reference criteria for a 3-DOF linear motion random MET.

$ASD_{xx}(f)$	$CSD^{*}_{xy}(f)$	$CSD^{*}_{xz}(f)$
$CSD_{xy}(f)$	$ASD_{yy}(f)$	$CSD^{*}_{yz}(f)$
$CSD_{xz}(f)$	$CSD_{yz}(f)$	$ASD_{zz}(f)$

Ideally, field measurements will be available to define both auto and cross spectral densities. One note regarding the development of vibration criteria for a Procedure II MET is that, unlike the SESA case, it is difficult to develop a composite set of reference spectra for a MEMA test. The difficulty lies primarily in the inability to characterize the CSD terms across an ensemble of measurements. This issue is discussed in further detail in Annex E.

2.1.1 Cross Spectral Density Structure.

Most of the commercially available MET control systems provide a method of entering the CSD terms in the form of relative phase and coherence. For example, if one wished to conduct a vertical only test using the two-exciter configuration illustrated in Figure 527.2-2, the ideal reference would be a phase setting of 0 degrees with a coherence of 1.0. Similarly, if the motion desired was pure pitch, the ideal reference would be a phase setting of 180 degrees with a coherence of 1.0. Unfortunately, selecting a coherence setting of 1.0 results in a singular SDM. Furthermore, it is very rare to find perfectly coherent measurements in practice due to noise and system non-linearities. Experience has shown that when specifying highly coherent measurements in a MET, a coherence selection that is slightly less than 1.0, (γ_{ij} = .95 to .98), greatly reduces the numerical concerns associated with a singular SDM, and the desired frequency and temporal characteristics are still achieved to a high degree.

Direct knowledge of the CSD characteristics of the field environment is desired as the phasing characteristics between mechanical DOF's may have a significant effect on the response of the UUT. Modal characteristics of the UUT may highly influence response dynamics as a function of the relative phasing of the reference (drive) signals.

2.2 Control Hierarchy.

In earlier MET control algorithms as discussed in paragraph 6.1, reference h, in the hierarchy of control for a MET, correction of the ASD terms were generally given priority. CSD terms were then corrected to the degree possible without corrupting the ASD terms. In modern MET algorithms, the drive signals are updated such that the SDM matrix has minimal mean-squared error. The degree of accuracy in replicating the CSD terms in a MEMA test are often test-specific, and associated tolerances should be tailored as appropriate. For example, consider a 6-DOF MET designed to address functional performance of a component such as a gimble-based stabilization platform for which one may have interest in the rotational degrees of freedom to a frequency that is much less than the full test bandwidth. For such cases, maintaining accurate CSD characteristics between control points will be predefined by the test performance objectives and the CSD characteristics at frequencies higher than the bandwidth of the required functional test are not considered critical.

2.2.1 Measured Data Available.

When in-service measurement data have been obtained, it is assumed that the data are processed in accordance with good data analysis procedures (see paragraph 6.1, references d and e). In particular, an adequate number of statistical degrees-of-freedom has been obtained to provide information with acceptable statistical error. Generally, careful attention must be given to the field measurement configuration. In particular, the location of the measurement points and qualification of the points as to whether they are structural points on the materiel capable of describing overall vibration characteristics of the materiel, or are response points on the materiel local to specific component response definition of the materiel. Consideration must be given to not only statistical error in auto-spectral density estimates, but also in cross-spectral density estimates (including transfer, coherence function estimates). For cross-spectral density transfer function estimates, it is important to correctly diagnose the coherence or lack of coherence among measurements. Ideally, the field and laboratory phase and coherence would match, implying an accurate match of boundary conditions. However, in practice this is rarely the case. Inspection of the field measured CSD terms is recommended to select key frequency bands in which one desires optimal coherence and phase matching as discussed in reference 19 of Annex E.

Low coherence implies that the vibration energy between measurements is uncorrelated, so that multiple exciters may be employed without cross-spectral information. Low coherence may also be viewed as a relaxation of strict cross-spectral information and perhaps use of the cross-spectral information that occurs naturally in the laboratory test configuration.

2.2.2 Measured Data Not Available.

When measurement data are not available and only specification level auto-spectral density information is available, it almost always needs to be assumed that excitation environments are independent of one another (coherence values are near zero). In addition, the effects of in-service and laboratory boundary condition impedance cannot be assessed. Normal mode information from the materiel is important in allowing the general decoupling of vibration modes of response. Careful attention must be given to the specification of the "control" and "monitoring" measurement points.

A control measurement point would typically be on a structural member and describe the overall vibration characteristics of the item. A monitoring measurement point would describe local vibration characteristics that are relevant for a specific component. Paragraph 6.1, reference j, provides information on extremes of excitation.

2.2.3 Use of 1-DOF References.

Employing highly conservative vibration specifications originally designed for a 1-DOF laboratory test as uncorrelated reference ASD's for a MDOF test should be addressed with caution. Vibration specifications developed for 1-DOF scenarios are often purposely conservative, in part to account for the fact that no significant coupling between mechanical DOF's is expected in the laboratory. However, such coupling between mechanical DOF's is certainly possible in the field or in a MDOF laboratory setting. Therefore, employing highly conservative spectra as references in a MDOF test could yield uncharacteristically high response in the event the unit under test has closely coupled structural modes between mechanical DOF's. If the conservatism characteristics of the 1-DOF references are clearly defined, it may be possible to develop an alternative set of uncorrelated references with reduced conservatism to address MDOF scenarios.

3. TEST TOLERANCES FOR A PROCEDURE II MET.

In general, all test tolerances need to be established based on some comparison in the frequency domain of the auto-spectral and cross-spectral density specifications with the corresponding laboratory test measured auto-spectral and cross-spectral information. Substantial reliance with respect to tolerances will be made on the auto-spectral density information, with cross-spectral density information playing a secondary role because of its reliance on measurement channel coherence for error characterization. Basic comparison might be taken for nominal test tolerances performed by the vendor-supplied MET software. Test laboratory personnel need to consult the vendor-supplied MET system manuals for such tolerances, and have a very clear understanding of the proper interpretation of the test tolerances. Unfortunately, the question of reasonable tolerances in a MET is not simple. Generally, the test tolerances prescribed in Method 514.8 for stationary random vibration are applicable for auto-spectral density information derived from a MET. However, it is often necessary to relax test tolerances on cross-spectral density information. Transfer function estimates along with coherence, partial coherence and multiple coherence function estimates may be necessary to assess the test tolerance questions. An experienced analyst will be required in cases where multi-channel measurements must be assessed for test tolerance assessment.

Since the test is run in real time, it is only necessary to ensure the reference input is properly compensated before running the test. All MET strategies and vendor software provide for very low level testing for establishing preliminary transfer function information that may be updated for higher level testing. The updated transfer function accounts for certain vibration system amplitude nonlinearities that may occur as the general level of vibration is increased.

3.1 Composite (Global) Error Discussion for Procedure II.

The same issues discussed in Annex C, paragraph 3.1, apply to Procedure II MET. However, for a Procedure II test, the time histories synthesized by the control system will be wide sense stationary and Gaussian in nature. Therefore, the global error discussion reduces to a discussion of the ASD and CSD error. Recall from the discussion in paragraph 2.2, that ASD is given priority in the control scheme, and that the degree of CSD accuracy required will be determined largely on a test-by-test basis. Addressing global error will depend largely on the MET configuration and control transducer placement. Translational and rotational degrees of freedom may be viewed in a composite sense by averaging or weighting each transducer in a common axis, or possibly by considering the composite ASD error across all axes as suggested in Annex C, paragraph 3.3. Translational degrees of freedom are readily computed from direct accelerometer measurements, while rotational degrees of freedom may be viewed in terms of the ASD computed from either direct angular motion measurements or from estimates of rotations computed from linear accelerometers. When considering estimates of rotational degrees of freedom based on linear accelerometers, refer to the guidance and caution discussed in Annex C, paragraph 2.2.

(This page is intentionally blank.)

METHOD 527.2, ANNEX E

LABORATORY VIBRATION TEST SCHEDULE DEVELOPMENT FOR MULTI-EXCITER APPLICATIONS

1. SCOPE.

This Annex presents considerations and techniques for developing Laboratory Vibration Test Schedules (LVTS) that can be utilized to simulate field vibration environments on a vibration table. Laboratory vibration tests are used extensively in lieu of more time-consuming and less cost effective field exposure tests. This Annex specifically addresses random vibration testing controlled to frequency-domain vibration spectra and is intended to address multiple "exciter" (also referred to as "shaker" or "actuator") scenarios with the emphasis on mechanical multiple degree-of-freedom (MDOF) scenarios. There is a significant increase in complexity between single-exciter/single-axis (SESA) and multiple-exciter/multiple-axis (MEMA) testing in terms of both mechanics and control. MEMA specific issues ranging from definitions and nomenclature consistency, to data analysis techniques, will be addressed.

2. FACILITIES AND INSTRUMENTATION.

2.1 Facilities.

The development of a LVTS will require access to the test item of interest (or a dynamically equivalent surrogate), access to the carrier vehicle, appropriately placed transducers, signal conditioning and data acquisition hardware, and a controlled environment for collecting input data (e.g., a road course for wheeled and/or tracked vehicles, waterway for watercraft, airspace for aircraft, rotorcraft, and/or spacecraft).

2.2 Instrumentation.

a. LVTSs are generally defined in terms of acceleration units. The transducer of choice for making acceleration measurements is an accelerometer. This Annex will address LVTS development in terms of acceleration.

b. It is strongly recommended that the same model of accelerometer and signal conditioning is employed at all instrumented locations to preserve phase characteristics during both the field acquisition and laboratory test phase of any MDOF test. Refer to the guidelines in Military Standard (MIL-STD)-810H[1*] and Institute of Environmental Sciences and Technology (IEST) Recommended Practice IEST-RP-DTE012.2[2] for recommended accuracy of the transducers and associated signal conditioning.

3. REQUIRED TEST CONDITIONS.

The primary function of Vibration Schedule Development (VSD) is to combine vibration measurements of numerous events that collectively represent an item's lifetime vibration exposure (or some predefined subset thereof) into a manageable set of LVTS representing the equivalent exposure. The most dynamically accurate method to reproduce the full exposure would be to sequentially vibrate the system to all the individual, uncompressed events representing its full lifecycle. However, such an approach is generally not feasible from both schedule and economic perspectives and some compromises must be made to realize the benefits of testing in the laboratory. Time compression techniques based on fatigue equivalency are typically employed such that vibration testing can be performed in a timely and economic manner. North Atlantic Treaty Organization (NATO) Allied Environmental Conditions Test Publication (AECTP) 240, Leaflet 2410[3] and Method 514.8 of Mil-Std-810H, provide general guidance for developing accurate representations, and issues that should be considered during the VSD process for the SESA scenario. This Annex expands upon the discussion in Leaflet 2410 to address the general multiple exciter test scenario. Discussions will be limited to random LVTS development. At the time of this publication, no commercially available multiple-input multiple-output (MIMO) solutions exist for swept narrowband random on random (NBROR) or sine-on-random (SOR) other than Procedure I - Time Waveform Replication based techniques.

3.1. Test Configurations.

The MIMO random vibration test problem can refer to several configurations. One configuration is multiple exciters driving a single test item in one axis. This configuration is often used for large test items too large for a single exciter. A second configuration is the excitation of a single test item with multiple exciters in more than one axis. Linear displacements along defined directions are referred to as translation degree-of-freedom (DOF) and angular displacements along those same directions are referred to as rotation DOFs. Up to six DOFs exist for a rigid body

*Superscript numbers correspond to those in Appendix E, References.

(i.e., X-, Y-, Z-translations and roll, pitch, yaw rotations). In some cases, additional DOFs can be excited due to deformations of the test article and/or testing an item with articulating components.

3.1.1 Basic Representation of a MIMO System.

All MIMO test systems are discussed using a common description in terms of matrix equations[2,4,5]. A simplified version of the general MIMO random vibration test problem can be generalized in Figure 1. The complete mechanical system is characterized by the power amplifiers and a system of several exciters, on which is mounted a single test article. The response of the test article is monitored by a vector of response channels (represented as {c}). Each element in the vector is typically the acceleration time history from a single accelerometer. Other types of sensors can be used, with attention paid to the nature of the measurements relative to the test item and other sensors. The power amplifiers are driven by a vector of electrical drives (represented as {d}), generated by a control system. Each element in the vector is a time history driving a single shaker. The control system monitors the response of the test item {c}, and attempts to produce drive signals {d}, such that the statistics of the control signals meet some criteria as specified in the test specifications.

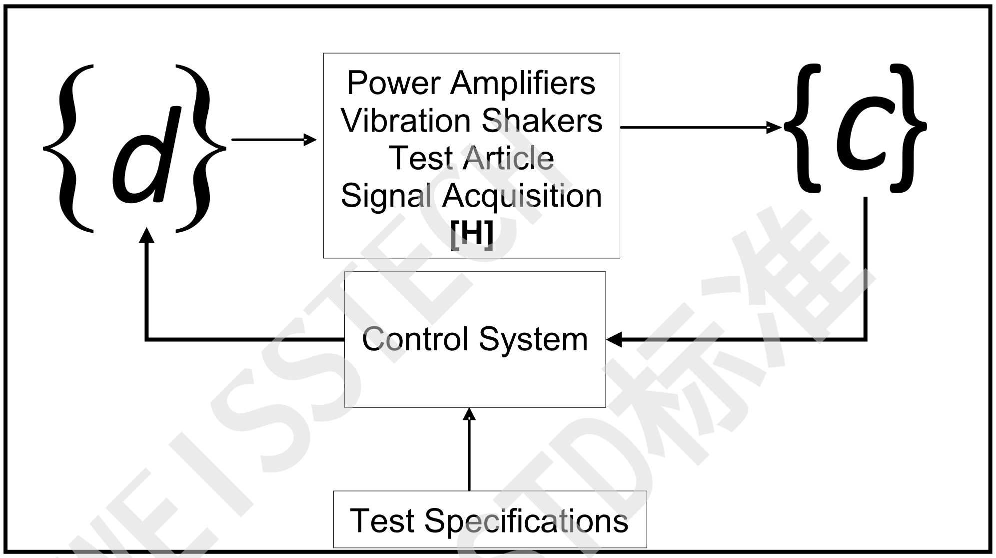


Figure 1. Basic representation of a MDOF system.

3.1.2 Generalized Representation of a MIMO System.

A more generalized MIMO system is shown in Figure 2. A system under test is driven by N_s shakers resulting in the response of N_a control accelerometers. The accelerometer data are typically structured in blocks. Each of the acceleration records will then be a vector of time samples. Some control systems then provide for a transformation matrix, $\mathbf{T_a}$, to convert the block of N_a accelerometer time histories to N_c control variables. The Spectral Density Matrix (SDM) of the control variables is then estimated from the current block of data and previous data. The transformation matrix, $\mathbf{T_a}$, is typically a constant independent of frequency. In theory the transformation matrix could be applied before or after the estimation of the control SDM. The estimated control SDM, **C** is then compared with the reference SDM, **R**, and a correction is computed for the drive SDM, **D**. The drive time histories {**d**} are then computed from the drive SDM, **D**, using time domain randomization. A second transformation matrix, $\mathbf{T_s}$, is employed to transform the N_d drive variables into N_s shaker drive signals. In theory, $\mathbf{T_s}$ could be implemented before or after the transformation into the time domain. One advantage of placing the transformation in the frequency domain section of the control algorithm is that the matrix could then be made a function of frequency. Having the transformation matrix, $\mathbf{T_s}$, a constant assumes the shakers are matched and the desired transformation can be deduced.

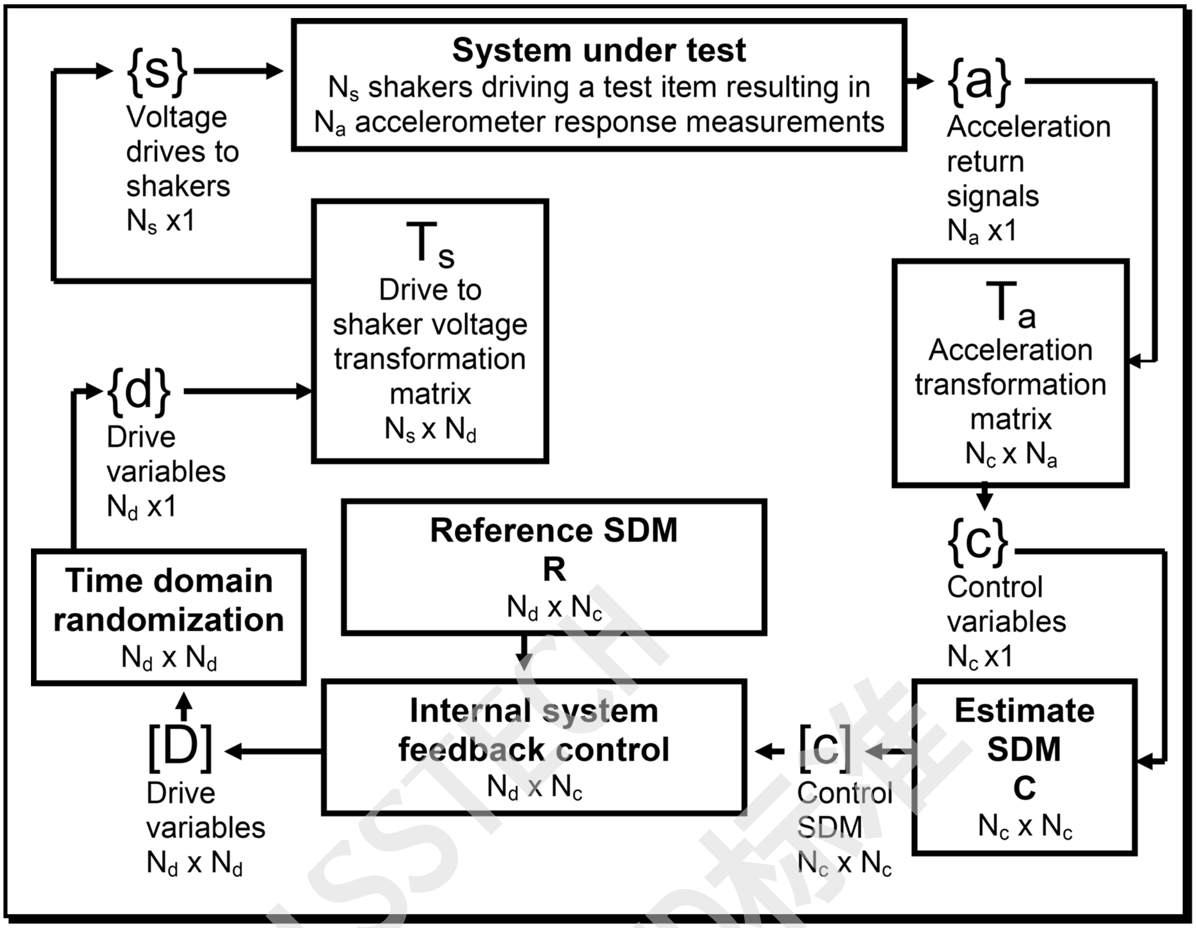


Figure 2. Generalized representation of a MDOF system.

3.2 Generalized MDOF Vibration Control Discussion.

a. A general discussion of the MDOF control process is provided for insight as to how the MDOF LVTS will serve as the reference in the control process. The purpose of the control loop is to minimize the difference between the reference and the control signals by making corrections to the drive signals. The correction can be computed in several ways. One method is to compute the drive from:

$$\mathbf{D} = \mathbf{Z}\hat{\mathbf{R}}\mathbf{Z}'$$

where the system impedance matrix, **Z**, is updated as new information is gathered, or a modified reference spectrum, $\hat{\mathbf{R}}$, is computed based on the error in the return spectrum. The initial drive vector is typically computed using the above equation and the reference SDM. A drive signal error can also be computed from:

$$\mathbf{D}_e = \mathbf{Z}(\mathbf{R} - \mathbf{C})\mathbf{Z}'$$

Sometimes an adaptive correction is used. Sometimes a combination of all methods is used.

b. The transformation matrices are often called the input and output transformation matrices. One should be careful with this nomenclature because of the confusion between input and output. The input to the system under test (voltages to the power amplifiers or servo controllers) is the output of the control system. The output of the system under test (such as accelerometer measurements) is the input to the control system. Paragraphs 4.4.1 and 4.4.2 provide the nomenclature employed for input and output transformations, as they are applied within this document.

c. Minor errors in the matching of shakers can be corrected by the control algorithm, but major mismatches could be problematic. The time domain drive signals (represented by {s}), are sent to the shakers completing the control loop.

d. If $\mathbf{T_a}$ is not available, then $N_a = N_c$ and {a} = {c}. If $\mathbf{T_s}$ is not available, then $N_d = N_s$ and {d} = {s}. If $N_d = N_c$, the number of control variables and the number of drive variables are the same. This is referred to as square control. Square control is the most common control method. If $N_s > N_a$ the system is over-actuated and least squares approach using a pseudo inverse (pinv) is typically used to determine the drive signals. If $N_s < N_a$ the system is under-actuated and exact control of the control SDM is often not possible. In such cases, some kind of average control is usually implemented. Often when $N_s \neq N_a$ some combination of the transformation matrices are often used to force square control, $N_d = N_c$.

e. The entire mechanical system can be characterized by a matrix of frequency response functions [**H**]. For the typical case, these frequency response functions will have units of g/V (acceleration in gravitational units/volts of drive). For the typical case, the control signals are characterized by a SDM. The diagonal elements are the autospectral density (ASD or PSD) of the control signals. The off diagonal elements are the cross spectral densities (CSD) between pairs of control signals. The input to the system is characterized by the SDM of the voltage drive signals. The fundamental relationship between the drives and the control signals is given by:

$$\mathbf{C} = \mathbf{H}\mathbf{D}\mathbf{H}'$$

f. The complex conjugate transpose is denoted by [ ]'. All of the matrices in the equation are complex functions of frequency. The spectral density matrix is Hermitian[6], i.e. $D_{ij} = D_{ji}{}^{*}$ where $D_{ji}{}^{*}$ is the complex conjugate of D_{ji}, and D_{ji} is an element from a spectral density matrix. Note that this requirement demands that the diagonal elements are real. Note that C and D are square matrices; they have the same number of rows and columns. C and D are the same size only if H is square, i.e. the same number of inputs and outputs. To be physically realizable, the SDM must also be positive semi-definite. This requirement will be discussed in paragraph 4.5.2.

g. The drive spectral density matrix is converted into the drive time histories using the method of time domain randomization[4]. The spectral density matrix is typically estimated using Welch's method[7].

4. TEST PROCEDURES.

VSD requires a thorough knowledge of the dynamic environment to which the test hardware will be exposed when fielded. This knowledge must include characterization of the exposure levels and durations for all relevant conditions.

4.1 Development of Mission or Lifetime Scenario.

The duration of the vibration environments can be derived from the item's Life Cycle Environment Profile (LCEP). The life cycle will include many different types of induced mechanical environments which may occur while the materiel is being handled, transported, deployed and operated. Although all the induced mechanical environments are not critical in terms of generating potential damaging response amplitudes, they contribute in varying degrees to the materiel's fatigue damage. All expected exposure conditions should be tabulated, along with corresponding durations, to form the items lifetime "scenario". The scenario is a key parameter in the development of any vibration schedule.

4.2 Limitations.

The mechanical degrees of freedom (DOFs) for which a VSD effort is capable of addressing, is a function of the number and placement of the transducers employed in the field data acquisition phase. Similarly, the maximum number of mechanical DOFs possible to reproduce in the laboratory environment is a function of the number and placement of actuators and coupling hardware. This Annex will consider the general case for VSD development in which the reference SDM will be defined in terms of the six classical (3-translational and 3-rotational) rigid body mechanical DOFs. In the event less than six mechanical DOFs are being considered, the generalized theory is easily configured to address the motion of interest.

4.3 Field Data Acquisition.

When in-service measurement data have been obtained, it is assumed that the data is processed in accordance with good data analysis procedures, as in Multi-Shaker Test and Control IEST-RP-DTE022.1[8] and Welch's method. In particular, an adequate number of statistical degrees of freedom (DOFs) have been obtained to provide information with acceptable statistical error. Consideration must be given to not only statistical error in auto-spectral density estimates, but also in cross-spectral density estimates (including transfer and coherence function estimates).

4.3.1 Instrumentation.

For the purpose of this Annex, all instrumentation related discussions will be limited to linear accelerometers and engineering units of g's, as was the case in the general control discussion provided in paragraph 3.1.1. Linear accelerometers have several advantages including familiarity to most users, low cost, wide bandwidth, small size and weight, and readily available low cost highly reliable signal conditioning options.

4.4 Use of Rigid Body Modes.

a. In single axis testing, the control input is often defined with a single accelerometer. This is satisfactory if the shaker and test fixtures are rigid within the frequency band of interest. If the shaker and test fixtures are not rigid, the technique of using a single accelerometer for control can sometimes lead to serious difficulty. To overcome these problems, methods using the average of several accelerometers and/or force limiting have come into common practice. In MEMA testing, the problem can be more serious as non-rigid body response is more common. When considering the special case of multiple shakers exciting a test item with multiple rigid body degrees of freedom, the use of the input transformation to define the response in terms of rigid body modes has several advantages. It is somewhat analogous to a generalization of the common practice for single axis testing. If there are more control channels than rigid body degrees of freedom, and an input transformation matrix is defined to transform the control accelerometers into rigid body modes, one essentially defines the motion of each rigid body mode as a weighted average of the accelerometers active for the mode. In many cases, given the control authority of the shakers, this is about the best viable solution. It is analogous to averaging accelerometers for a single axis test, which is common practice. The elastic modes are not controlled, since often the control authority over these modes does not exist. The system is driven with an equivalent rigid body motion in each of the rigid body modes. It is necessary to make sure that for any mode the transformation of the control accelerometers $\{a\}$ does not result in zero for any of the rigid body modes. If higher flexural modes are present they will not be controlled. In theory the flexural modes can be limited by adding control variables, but this requires knowledge of the modes in the test setup. This information can only be determined with materiel in the test configuration. For this reason, it is sometimes desirable to allow modification of the test requirements after this information is made available. Exactly how this will be accomplished in specification writing will have to be determined at a later date.

b. An advantage of using rigid body modes in the specification is that the field measurements used to define the environment can be made with the transducers in locations different from the locations of the transducers used in the laboratory test. The field measurements are reduced to equivalent rigid body modes using an acceleration transformation matrix (refer to paragraph 4.4.1), and the modes are controlled on the test using another transformation matrix for the laboratory test configuration. The two transformation matrices do not have to be the same. Use of alternate control points, while maintaining a full rank transformation matrix, provides a way of making the laboratory test "equivalent" in the sense of the rigid body modes.

c. A practical difficulty arises when more modes are attempted to be controlled. The general case of six (6) rigid body modes requires the specification of a 6 x 6 SDM (6 ASD's and 15 CSD's). Physical understanding of the SDM matrix associated with rigid-body motion by itself is difficult without the additional complications of elastic DOFs. Furthermore, it is difficult to assure that the specification results in a positive definite SDM, which is a physical requirement. (Additional discussion on positive definite matrices is the subject of paragraph 4.5.2.)

4.4.1 Acceleration (Input) Transformation.

The acceleration to control space transformation matrix, $\mathbf{T_a}$, commonly referred to as the "input transformation matrix" from the control system perspective, is defined in the article "Applying Coordinate Transformations to Multi-DOF Shaker Control"[9] and generalized in the article "Benefits and Challenges of Over-Actuated Excitation Systems"[10]. The acceleration transformation matrix transforms a set of accelerometer measurements into a set of control variables. Often these control variables are descriptions of rigid body modes. The acceleration transformation is usually performed in the time domain as:

$$\{\mathbf{c}\} = \mathbf{T}_{\mathbf{a}}\,\{\mathbf{a}\}$$

4.4.1.1 Acceleration (Input) Transformation Derivation.

One goal of this Annex is to define a standard nomenclature. The following summary has been restructured to the nomenclature defined by this Annex. Referring to the input transformation derivation[10], a generic acceleration measurement at the k^{th} position in orientation j is structured as Equation 4.1:

$$a_{k_j} = \begin{bmatrix} \underline{e}_j^T & -\underline{e}_j^T \left[{}^{P}\underline{r}_i^{P} \right]^{\times} \end{bmatrix} \begin{bmatrix} {}^{P}\underline{a}_o^{P} \\ {}^{P}\underline{\alpha}^{P} \end{bmatrix} \tag{4.1}$$

where a_0 is the linear acceleration at some reference point designated the "origin", α is the angular acceleration of the body (assuming it is rigid), $k \in (1, 2, ..., N_a)$, $i \in (1, 2, ..., n^*)$, $j \in (x, y, z)$, and $\underline{e}_x^T = [1\ 0\ 0]$, $\underline{e}_y^T = [0\ 1\ 0]$, and $\underline{e}_z^T = [0\ 0\ 1]$ are row selection vectors (as shown assuming accelerometer orientation is aligned per a traditional right hand Cartesian system). Parameter N_a represents the number of accelerometer measurements (as previously defined) and $n^* \le N_a$ the number of measurement locations; e.g., utilization of multi-axis accelerometers results in $n^* < N_a$. Vector r_i is the position vector relating the position of measurement location i to a user defined origin. $\left[{}^{P}\underline{r}_i^{P} \right]^{\times}$ is the skew symmetric operator equivalent of the cross product, making the matrix based computations in Equation 4.1 possible. The matrix equivalent of a vector (i.e., a coordinatized vector quantity) is denoted as ${}^{(\,)}\left(\underline{\;\;}\right)^{(\,)}_{(\,)}$ where the right superscript and subscript identify the body and point of interest respectively, and the left superscript denotes the coordinate frame in which the vector quantity was coordinatized; e.g., ${}^{P}\underline{r}_i^{P}$ in Equation 4.1 denotes the ith point on body P (the platform) coordinatized in frame $\mathcal{F}_P$ - the platform's coordinate frame.

4.4.1.2 Equation 4.1.

Equation 4.1 represents one equation in six unknowns, the three components of the linear acceleration of the reference point and the three components of the rigid body angular acceleration. In order to determine these quantities, at least six measurements are needed. These requirements are not as stringent as that reported in the article "On the Use of Linear Accelerometers in Six-DOF Laboratory Motion Replication"[11] because of the assumptions above (i.e., small angular velocities and rigid body).

Let’s consider the most general case of N_a measurements from n* locations. In this case, Equation 4.1 becomes:

$$\left[a_{k_j}\right] = \underset{(n\times 1)}{\begin{bmatrix} a_{1_j} \\ a_{2_j} \\ \vdots \\ a_{n_j} \end{bmatrix}} = \underset{(n\times 6)}{\begin{bmatrix} \underline{e}_j^T & -\underline{e}_j^T\left[{}^P\underline{r}_1^P\right]^{\times} \\ \underline{e}_j^T & -\underline{e}_j^T\left[{}^P\underline{r}_i^P\right]^{\times} \\ \vdots & \vdots \\ \underline{e}_j^T & -\underline{e}_j^T\left[{}^P\underline{r}_{n^*}^P\right]^{\times} \end{bmatrix}} \underset{(6\times 1)}{\begin{bmatrix} {}^P\underline{a}_o^P \\ {}^P\underline{\alpha}^P \end{bmatrix}}, \; i\in\left(1, 2, \ldots, n^*\right), \; j\in\left(x, y, z\right)$$

which using the nomenclature defined in this Annex is of the form:

$$\underset{(\mathbf{n}\times\mathbf{1})}{\left\{\mathbf{a}\right\}_{\mathbf{Meas}}} = \underset{(\mathbf{n}\times\mathbf{6})}{\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]} \underset{(\mathbf{6}\times\mathbf{1})}{\left\{\mathbf{c}\right\}_{\mathbf{Motion}}} \tag{4.2}$$

where $\left\{\mathbf{c}\right\}_{\mathbf{Motion}}$ is a 6 x 1 matrix of unknown linear and angular accelerations and $\left\{\mathbf{a}\right\}_{\mathbf{Meas}}$ is an nx1 matrix of acceleration measurements. Observe that $\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]$ is entirely defined by knowledge of (i) placement, (ii) orientation, and (iii) utilized signals of the accelerometers.

Observe that if $\overline{\mathbf{T}}_{\mathbf{a}}$ is of full column rank, then $\left[\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\right]^{\mathbf{-1}}$ exists enabling $\left\{\mathbf{c}\right\}_{\mathbf{Motion}}$ to be solved as follows:

$$\begin{aligned}
\left\{\mathbf{a}\right\}_{\mathbf{Meas}} &= \overline{\mathbf{T}}_{\mathbf{a}}\left\{\mathbf{c}\right\}_{\mathbf{Motion}} \\
\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\left\{\mathbf{a}\right\}_{\mathbf{Meas}} &= \overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\left\{\mathbf{c}\right\}_{\mathbf{Motion}} \\
\left[\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\right]^{\mathbf{-1}}\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\left\{\mathbf{a}\right\}_{\mathbf{Meas}} &= \left[\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\right]^{\mathbf{-1}}\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\left\{\mathbf{c}\right\}_{\mathbf{Motion}} \\
\left[\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\right]^{\mathbf{-1}}\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\left\{\mathbf{a}\right\}_{\mathbf{Meas}} &= \left\{\mathbf{c}\right\}_{\mathbf{Motion}}
\end{aligned}$$

Defining $\mathbf{T}_{\mathbf{a}} \equiv \left[\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\right]^{\mathbf{-1}}\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}$, Equation 4.2 can be rewritten as:

$$\left\{\mathbf{c}\right\}_{\mathbf{Motion}} = \left[\mathbf{T}_{\mathbf{a}}\right]\left\{\mathbf{a}\right\}_{\mathbf{Meas}} \tag{4.3}$$

Where $\left[\mathbf{T}_{\mathbf{a}}\right]$ is a 6 x n matrix referred to in the literature as the “Acceleration Transform Matrix” or “Input Transform Matrix”. Observe that the critical requirement that $\left[\overline{\mathbf{T}}_{\mathbf{a}}^{\mathbf{T}}\overline{\mathbf{T}}_{\mathbf{a}}\right]^{\mathbf{-1}}$ exists in order to derive the input transformation matrix $\left[\mathbf{T}_{\mathbf{a}}\right]$, is solely a function of placement and orientation of measurement transducers.

4.4.2 Drive (Output) Transformation.

a. Although details of the Drive Transformation are not required to develop a MDOF VSD reference, a short summary of the concept is provided for general knowledge. Referring to the schematic in Figure 2, transformation matrix $\mathbf{T_s}$ transforms the $\mathrm{N_d}$ drive variables into $\mathrm{N_s}$ shaker drive signals. Reference 10 provides a formal derivation of the transformation matrix, $\mathbf{T_s}$. Note that while the “acceleration transformation” was computed based on knowledge of position and polarity of the control accelerometers, the transformation matrix, $\mathbf{T_s}$ is dependent upon the position and line of action (LOA) of the individual actuators. In this Annex and within reference 10 $\mathbf{T_s}$ is referred to as the “drive transformation” or “output transformation”. The following cases summarize the computation of $\mathbf{T_s}$ and the effect on the control process.

(1) Case 1: Configurations in which the number of motion degrees-of-freedom or control signals, N_c and the number of output control variables, N_d are the same is referred to as “square” control. If the number of output control variables, N_d and the number of shakers, N_s is the same, the transformation matrix, $\mathbf{T_s}$ will simply be the Identity matrix.

(2) Case 2: Configurations in which the number of shakers N_s exceeds the number of output control variables N_d, the excitation system is said to be over-determined or over-actuated. In such cases, some of the drives will be linear combinations of other drives. Furthermore, if $\mathbf{T_s}$ is a constant which is employed in the time domain, the individual actuators must be matched (e.g. matched frequency response functions (FRFs)).

(3) Case 3: Configurations in which the number of shakers, N_s is less than the number of control signals, N_c, the excitation system is said to be under-determined or under-actuated. In such cases, exact control of the SDM is not possible.

b. In theory, $\mathbf{T_s}$ could be implemented before or after the transformation into the time domain. One advantage of placing the transformation in the frequency domain section of the control algorithm is that the matrix could then be made a function of frequency. Having the transformation matrix, $\mathbf{T_s}$, a constant assumes the shakers are matched and the desired transformation can be deduced.

4.4.2.1 Drive (Output) Transformation Derivation.

a. As previously stated, one goal of this Annex is to recommend a standard nomenclature. The following summary from reference number 10 has been restructured to the nomenclature recommended by this Annex. Figure 3 illustrates the generalized multi-axis vibration system.

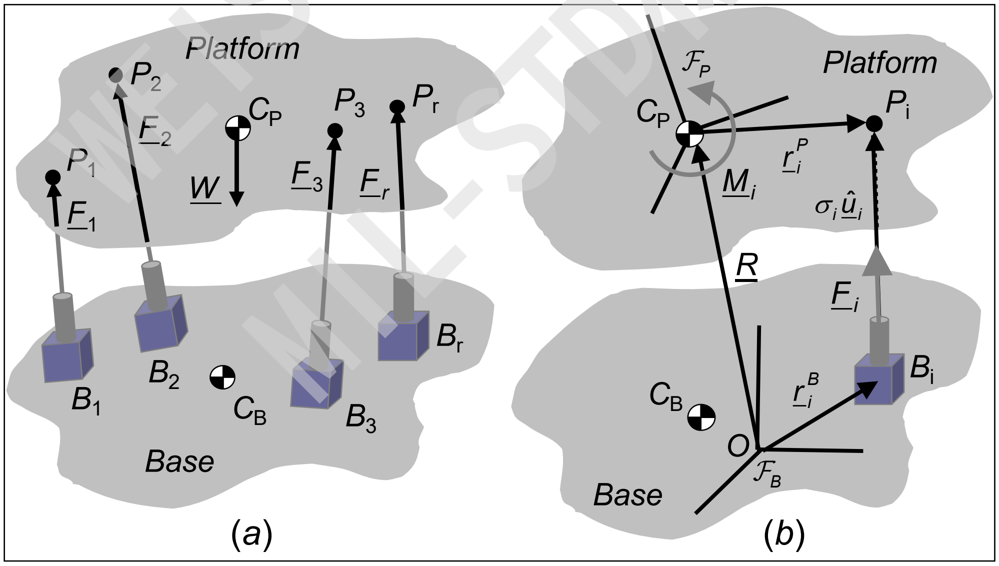


Figure 3. Generalized multi-axis vibration system.

b. Refer to reference number 10 for a detailed derivation of Equation 4.4. The following summary illustrates how the output transform, $\mathbf{T_s}$ is associated with the P-Matrix, (Plucker Matrix) discussed in the reference.

$$\begin{bmatrix} {}^{B}\underline{\hat{u}}_1 & {}^{B}\underline{\hat{u}}_2 & \cdots & {}^{B}\underline{\hat{u}}_{N_s} \\ {}^{P}\underline{m}_1^{P} & {}^{P}\underline{m}_2^{P} & \cdots & {}^{P}\underline{m}_{N_s}^{P} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{N_s} \end{bmatrix} = \begin{bmatrix} m\left({}^{B}\underline{a}_C^{P} - {}^{B}\underline{g}\right) - {}^{B}\underline{F}_E \\ {}^{P}\underline{\underline{I}}_C^{P}\,{}^{P}\underline{\dot{\omega}}^{P} + \left[\underline{\omega}^{P}\right]^{\times}\,{}^{P}\underline{\underline{I}}_C^{P}\left({}^{P}\underline{\omega}^{P}\right) - {}^{P}\underline{M}_E \end{bmatrix} \tag{4.4}$$

$$\underset{6xN_s}{\mathbf{P}} \qquad \underset{N_s x1}{\mathbf{F}} \qquad \underset{6x1}{\mathbf{C}}$$

c. In Equation 4.4, $[\mathbf{P}]$ represents the Plucker Matrix which is derived from known geometric parameters associated with the individual actuators, $[\mathbf{F}]$ represents the drive and $[\mathbf{C}]$ represents the desired motion. The variables ${}^{B}\underline{\hat{u}}_i$ represent the LOA vectors for each of the actuators and ${}^{P}\underline{m}_i^{P}$ is the moment arm associated with force f_i. Observe that the maximum dimension for the $[\mathbf{C}]$ matrix will be six, if all six traditional motion DOFs are being considered (i.e. $N_d = 6$). As stated in paragraph 4.4.2, Case 1 scenarios will simply have an identity matrix as the output transformation matrix and Case 3 scenarios (under-actuated) will not have a unique solution. Case 2 scenarios (over-actuated) may be addressed in terms of output transformations. The objective is to determine $[\mathbf{F}]$ in Equation 4.4, yielding the N_s drive signals as follows:

(1) Define $\underset{N_s x1}{\mathbf{F}} \equiv \underset{N_s x6}{\mathbf{P}}^{T} \underset{6x1}{\mathbf{D}}$ and substitute into $\underset{6xN_s}{\mathbf{P}} \underset{N_s x1}{\mathbf{F}} = \underset{6x1}{\mathbf{C}}$ yielding $\underset{6xN_s}{\mathbf{P}} \underset{N_s x6}{\mathbf{P}}^{T} \underset{6x1}{\mathbf{D}} = \underset{6x1}{\mathbf{C}}$

(2) $\underset{6xN_s}{\mathbf{P}} \underset{N_s x6}{\mathbf{P}}^{T}$ will be of full rank (i.e. invertible) if $[\mathbf{P}]$ is of full rank.

(a) If $[\mathbf{P}]$ is of full rank: $\underset{6x1}{\mathbf{D}} = \left[\underset{6xN_s}{\mathbf{P}} \underset{N_s x6}{\mathbf{P}}^{T}\right]^{-1} \underset{6x1}{\mathbf{C}}$

(b) If $[\mathbf{P}]$ is not full rank, actuator placement is not sufficient to obtain the mechanical DOF’s desired.

(3) Substituting results from (2) yields $\underset{N_s x1}{\mathbf{F}} \equiv \underset{N_s x6}{\mathbf{P}}^{T} \underset{6x1}{\mathbf{D}} = \underset{N_s x6}{\mathbf{P}}^{T} \left[\underset{6xN_s}{\mathbf{P}} \underset{N_s x6}{\mathbf{P}}^{T}\right]^{-1} \underset{6x1}{\mathbf{C}}$

(4) $\underset{N_s x6}{\mathbf{T}_s} \equiv \underset{N_s x6}{\mathbf{P}}^{T} \left[\underset{6xN_s}{\mathbf{P}} \underset{N_s x6}{\mathbf{P}}^{T}\right]^{-1}$

d. The discussions within this paragraph and previous derivation assumed $N_d = 6$. In the event $N_d < 6$, N_d would represent the actual number of mechanical DOFs. In terms of the nomenclature of Figure 2, and assuming matched actuators are employed, voltage drives to the shakers for the over-actuated scenario would be defined as Equation 4.5:

$$\{\mathbf{s}\} = \mathbf{T}_s\{\mathbf{d}\} \tag{4.5}$$

4.5 Data Analysis.

a. Ensure transducer placements have been addressed, to guarantee the desired motion DOFs may be resolved (refer to paragraph 4.4.1.2), and that common data validity checks are performed. Then, it is recommended that appropriate combinations of the linear acceleration measurements be transformed into the desired traditional motion DOFs through implementation of the acceleration transformation matrix. The transformed time histories will be referenced to a single point on the structure referred to as the “origin” as discussed in paragraph4.4.1.

b. A SDM for each test configuration identified in the mission scenario should be computed. In addressing the VSD techniques for reducing an ensemble of data, in this case an ensemble of SDM’s, the analyst will be required to deal with the ASD terms (the diagonal terms of the SDM) and CSD terms (the off-diagonal terms of the SDM).

4.5.1 Phase and Coherence Based Representations of CSD Terms.

Although the off-diagonal terms of the SDM are computed in terms of a CSD, it is common among control system vendors to allow cross terms to be defined in terms of Phase and Coherence. This is a convenient option in that it is often easier to physically interpret SDM CSD terms in terms of Phase and Coherence. There is a direct relationship between the two techniques of defining the cross terms of the SDM that is dependent upon the definition of ordinary coherence between two signals, $\gamma_{ij}^2 = \frac{|G_{ij}|^2}{G_{ii}G_{jj}}$. Normalizing the CSD terms of the SDM by $\sqrt{G_{ii}G_{jj}}$ yields a normalized spectral density matrix ($\mathrm{SDM_n}$) in which the ASD terms are not affected and the magnitude of the normalized CSD terms are defined as $\frac{G_{ij}}{\sqrt{G_{ii}G_{jj}}}$, which is equivalent to the square root of the ordinary coherence function, while not affecting the original phase relationship of the CSD terms. Similarly, the normalized spectral density matrix, $\mathrm{SDM_n}$, may be transformed back to the original CSD form of the SDM.

4.5.2 Positive Definite SDM Considerations.

a. Any specified spectral density matrix must be positive semi-definite to be physically realizable. In practice it must be positive definite. The determinate of the matrix must be ≥0. All the eigenvalues of the SDM must be ≥0. This must be true at all frequencies. It must be possible to perform a Cholesky decomposition of the specified SDM. Another property of positive semi definite matrices is from Matrix Computations[12]:

$$|\Phi_{ij}|^2 \le \Phi_{ii}\Phi_{jj} \quad or \quad 0 \le \gamma^2 = \frac{|\Phi_{ij}|^2}{\Phi_{ii}\Phi_{jj}} \le 1$$

In the terms of random vibrations the ordinary coherence, γ^2 between signals must be less than or equal to one. In practical terms, if the coherence between any pair of signals is one, the SDM will be positive semi-definite and the control system will have problems. Note that in general, if **D** is Hermitian and positive semi-definite **C** will also be Hermitian and positive semi-definite.

b. If all the eigenvalues are non-negative, the matrix is positive semi-definite. If any of the eigenvalues are zero, it implies that one or more of the rows of the spectral density matrix are a linear combination of other rows. In practice, one would typically expect to deal only with positive definite matrices. Observe that even a small amount of noise or nonlinearity will result in a positive definite matrix. If a matrix is positive definite, the matrix can always be factored using Cholesky decomposition,

$$\mathbf{\Phi} = \mathbf{LL'}$$

where **L** is a lower triangular matrix. Which without loss of generality can be rewritten as,

$$\mathbf{\Phi} = \mathbf{LIL'}$$

where **I** is the identity matrix. In this application, **I** is not really the identity matrix. **I** is a spectral density matrix. At every frequency, **I** is a diagonal matrix of ones. The components in **I** are independent since all the off diagonal elements are zero. It is now clear why the cross spectral density matrix must be positive definite. If any of the elements in **I** are zero, it implies that there are less than N (the number of rows or columns in $\mathbf{\Phi}$) independent sources in $\mathbf{\Phi}$. Some of the rows and columns are linear combinations of other rows and columns. The identity matrix is positive definite, therefore $\mathbf{\Phi}$ must be positive definite. Using the interpretation of Random Data Analysis and Measurement Procedures[13], the diagonal elements of **I** can be interpreted as the auto-spectral densities of independent random noise sources. The maximum number of independent noise sources is N. If some of the elements in **I** are zero, the problem can still be solved by making the corresponding rows and columns of **L** zero. This is the positive semi-definite case. This case corresponds to the case where there exists less than N independent sources. Some of the N sources are linear combinations of other sources. This case will be very difficult for the control system. In general one may make some of the sources small but not zero. Part of this document will discuss the generation of a desired control SDM to make the control problem achievable and hopefully relatively easy for the control system to implement.

c. In general the control problem is an inverse problem. The desired control SDM (the output of the system under test) is known, and the drive (input to the system under test) SDM must be computed. There is a potential point of confusion here. The control system manufacturers treat the drive SDM as the output of the control system, which is the input to the shaker system. Similarly, the control system input is the output of the shaker system. Paragraphs 4.4.1 and 4.4.2 provide nomenclature employed for input and output transformations as they are applied within this document.

d. Inverse problems can be very difficult as multiplication by a matrix inverse is required. If the matrix is ill-conditioned, the result will be similar to dividing by zero for the scalar case.

For the case in which the number of inputs and outputs are the same; **H** is a square matrix of FRF's. The solution is to invert **H**. The solution for the drive matrix is then given by:

$$\mathbf{Z} = \mathbf{H}^{-1}$$

$$\mathbf{D} = \mathbf{Z}\mathbf{R}\mathbf{Z}'$$

This of course assumes **H** is well conditioned and the inverse exists. Part of this document will discuss issues to help the process of achieving a well conditioned **H** matrix.

The **H** matrix is typically estimated from:

$$\hat{\mathbf{H}} = \hat{\mathbf{S}}_{\mathbf{CD}}\hat{\mathbf{D}}^{-1}$$

The inverse of $\hat{\mathbf{D}}$ must exist. This implies that $\hat{\mathbf{D}}$ must be positive definite. The initial estimate of **H** is determined by exciting the system with a set of independent white inputs in a pretest environment. If **H** is to be corrected during the test, $\hat{\mathbf{D}}$ must be positive definite during the test or special provisions must be used to avoid the inversion of $\hat{\mathbf{D}}$ at frequencies where $\hat{\mathbf{D}}$ is not positive definite. This is one of the reasons the reference **R** rarely has any of the coherences equal to unity.

4.5.3 Data Compression.

a. Use of time compression techniques such as Miner-Palmgren may be employed to modify the ASD terms. References numbers 1 and 3 provide discussions on time compression. In the simplest terms, the Miner-Palmgren Hypothesis (Miner's rule) is a set of mathematical equations used to scale vibration spectra levels and their associated test times. It provides a convenient means to analyze fatigue damage resulting from cyclical stressing. The mathematical expression and variable descriptions for this technique are illustrated in Equation 4.6:

$$\frac{t_2}{t_1} = \left[\frac{S_1}{S_2}\right]^M \tag{4.6}$$

where:

t_1 = equivalent test time

t_2 = in-service time for specified condition

S_1 = severity (root mean square ((rms)) at test condition

S_2 = severity (rms) at in-service condition

(The ratio S_1/S_2 is commonly known as the exaggeration factor.)

M = a value based on (but not equal to) the slope of the S-N curve for the appropriate material where S represents the stress amplitude and N represents the mean number of constant amplitude load applications expected to cause failure. For the MDOF VSD work at hand, the default of $M = 7$ was selected per reference number 1.

b. It is recommended that the final vibration specification ASD terms are no greater than 3 decibel (dB) higher than maximum values measured in the field. Miner-Palmgren will be employed to the ASD portion of the SDM in the same manner as one would employ for a traditional 1-DOF scenario. Details such as maintain common test durations between mechanical DOFs are addressed in Paragraph 6.

4.5.4 Limiting Strategies.

Traditional notching techniques may also be employed if impedance mismatches lead to unrealistically high test item response. Notching techniques may be employed across all actuators with equal weighting or by weighting notching at each actuator as a function of coherence between the actuators and the location of interest. In addition to traditional notching based on acceleration spectra, it is also possible to consider limiting based on other parameters (e.g. von Mises Stress or Force limiting). As with any notching scheme, it is critical that any resulting deviations to the test or test tolerances must be approved by the appropriate test authority and must be clearly documented in the test plan and final report.

4.5.5 Minimum Drive Considerations.

A number of challenges have been identified in addressing the objective of establishing a reference SDM for multiple exciter test (MET) scenarios. One major area of concern is related to the fact that it is highly likely that there will be mechanical impedance differences between the field and laboratory conditions. Given these impendence mismatch issues, it is undesirable to force the test item into what could potentially be an unnatural state as fixtured in the laboratory. Optimally, achieving the specified autospectra without excessively taxing the excitation system is desired. Smallwood made a general approach to establishing minimum drive criteria in the article “MIMO Linear Systems Extreme Inputs/Outputs”[14]. Unfortunately, the technique does not always guarantee the resulting SDM to be positive semi-definite.

4.5.5.1 Independent Drives.

a. Although an active area of research, general techniques to address minimum drive criteria have not been formally established at the time of this publication. A proposed approach for trending drive voltages towards minimums while maintaining a positive- definite SDM, is discussed in the article “A Proposed Method to Generate a Spectral Density Matrix for a MIMO Vibration Test”[15], and is summarized below:

(1) Taking a clue from the modal test community, assume the drive signals to the excitation system will be uncorrelated. Typically for a vibration test, the drives are the voltage inputs to the shakers. For a simulation, the inputs into a model are often forces. It is always possible to excite the system with uncorrelated inputs. This is standard practice in the modal community, and is standard practice when performing the system identification for MIMO test systems. This leads to the logical question: Is it possible to generate a set of uncorrelated inputs that will produce a desired set of response autospectra (the diagonal of the output SDM)?

(2) The general equation relating the control point accelerations to the drive voltages is given in Random Vibrations, Theory and Practice[16]:

$$\mathbf{S_Y} = \mathbf{H S_X H'}$$

where $\mathbf{H'}$ is the conjugate transpose of $\mathbf{H}$, and $\mathbf{S_X}$ and $\mathbf{S_Y}$ are SDM’s. $\mathbf{H}$ is a matrix of frequency response functions relating the output to the input of the excitation system. In our case, ideally, $\mathbf{S_X}$ will be a diagonal matrix. Let $\overline{\mathbf{X}}$ be a column vector of the diagonal of $\mathbf{S_X}$ or, $\overline{\mathbf{X}} = \mathrm{diag}(\mathbf{S_X})$, and

$\overline{\mathbf{Y}} = \mathrm{diag}(\mathbf{S}_{\mathbf{Y}})$. The relationship between the autospectra, as shown in Appendix D proof 1, is given by:

$$\overline{\mathbf{Y}} = \overline{\mathbf{H}}\overline{\mathbf{X}}$$

where:

$$\overline{\mathbf{H}} = \mathbf{H}.*\,\mathrm{conj}(\mathbf{H})$$

where: * indicates an element by element multiplication. $\overline{H}_{ij} = |H_{ij}|^2$.

The solution is given by:

$$\overline{\mathbf{X}} = \overline{\mathbf{H}}^{-1}\overline{\mathbf{Y}}$$

b. In some cases the result will include negative elements in $\overline{\mathbf{X}}$. This is not physically possible. It indicates that the desired ASD's cannot be achieved with independent drives. In this case the negative values are set to zero, and the output SDM is recomputed from $\mathbf{S}_{\mathbf{Y}} = \mathbf{H}\mathbf{S}_{\mathbf{X}}\mathbf{H'}$ using the modified input spectral density matrix (the negative values set to zero). The resulting control point acceleration autospectra, will not be at the desired levels. To correct this problem, the control point acceleration autospectra are rescaled to the desired levels, keeping the phase and ordinary coherence the same. This is accomplished by pre and post multiplying the SDM by a diagonal matrix whose elements are the square root of the ratio of the desired ASD to the computed ASD:

$$\mathbf{S}_{\mathbf{Ynew}} = \mathbf{S}_{s}\mathbf{S}_{\mathbf{Yold}}\mathbf{S}_{s}$$

where $\mathbf{S_s}$ is a diagonal matrix and:

$$S_{s,ii} = \sqrt{\frac{Y_{ii,new}}{Y_{ii,old}}}$$

Note: Setting $S_{Yii,new}$=1, provides an efficient way to compute the normalized SDM where the diagonals are one and the magnitude of the off diagonals squared are the ordinary coherence and the phase of the off diagonal elements is the phase of the cross spectra.

The drive SDM can then be computed as:

$$\mathbf{S}_{\mathbf{Xnew}} = \mathbf{Z}\mathbf{S}_{\mathbf{Ynew}}\mathbf{Z'}$$

where $\mathbf{Z} = \mathrm{pinv}(\mathbf{H})$, the Moore-Penrose pseudo inverse. If **H** is square and full rank, the solution typically ends here. If **H** is not square or not full ranked:

$$\mathbf{S}_{\mathbf{Y}new} = \mathbf{H}\mathbf{S}_{\mathbf{X}new}\mathbf{H'}$$

The $\mathrm{diag}(\mathbf{S}_{\mathbf{Y}new})$ may not yield the desired ASD's. In this case, an iterative approach will often improve the result.

4.6 Independent References.

a. It is sometimes desirable to define the reference spectrum in terms of a diagonal matrix of autospectra. Several reasons drive us in this direction. One common case is that only knowledge of the autospectra from the field environments is available. Several factors can result in this situation. First the field data may have been acquired without phase information. Second, the resulting cross spectra can have a very complicated structure which is impractical to implement in a specification. Enveloping amplitudes is possible, but enveloping the phase is much more difficult. Third, the specification may be a composite of several environments, making the definition of cross spectra very difficult. Fourth, the vehicle on which the field data were taken may not be identical to the test vehicle. Fifth, the boundary conditions in the field may be different from the boundary conditions in the laboratory.

b. Small changes in the modal frequencies caused by any of the above factors can change the phase at any frequency near a mode by a large amount. All these factors make the specification of the cross spectra difficult. An option is to ignore the cross spectra and set them all to zero. This has the theoretical advantage of providing an excitation that in some sense covers the control variable response space.

c. The drive signals can readily be computed yielding uncorrelated motion (in this case the SDM of the uncorrelated reference spectra **Y** is diagonal) from:

$$\mathbf{S}_{\mathbf{X0}} = \mathbf{Z}\mathbf{S}_{\mathbf{Y0}}\mathbf{Z'}$$

This approach is currently available in commercial control systems. You simply specify the reference SDM as a diagonal matrix with the cross spectra (or equivalently the coherences) zero or near zero. This is typically a conservative approach.

d. In contrast to the independent drive discussion in paragraph 4.5.5.1, the danger with the independent reference concept is that this specification of control variables may be overly conservative near frequencies dominated by a single mode. An important clue that the result may be overly conservative is the trace of the drive voltages. This trace should be monitored and if overly large in some band of frequencies, limits can be negotiated and implemented.

4.7 Recommended Practices Summary.

The following list provides recommendations and general guidance to be considered when addressing the multi-axis VSD.

a. If possible, specify the test in terms of the rigid body motion.

b. Over specification of the control accelerometers is desirable. Use more control accelerometers than degrees of freedom in the test.

c. If possible, the entire SDM should be specified. A method to automate the generation of envelopes may be desired. This will permit the generation of the envelopes to be less developer specific.

d. If the entire SDM is specified, it is suggested that the coherence be set to near zero if the desired coherence is below 0.2. It should be recognized that the estimation of coherence is a biased result (the result will always be positive). It is recognized that the estimated coherence will never be zero; however, the control software can attempt to make the coherence as low as possible. The tolerance on the coherence must recognize the bias. If the coherence is small the phase is not important. For convenience, establishing a zero phase is a reasonable specification when the coherence is low.

e. If step c becomes too complicated, it is recommended that the test be run with near zero coherence.

f. If step e results in unrealistic responses, try using the independent drive option.

g. Consider a compromise position between independent reference criteria of step e and independent drive criteria as recommended in step f.

h. If the drive requirements are excessive at some frequencies, allow the test to be modified to reduce the drive requirements as discussed in paragraphs 4.5.4 and 4.5.5.

i. It is understood that MIMO testing is more complicated than single-input single output (SISO) testing. The specifications must reflect the desires of a knowledgeable environmental test engineer. Good communication between the project team, the environmental test engineer and the test lab must be maintained to achieve the desired test results.

5. DATA REQUIRED.

As discussed in NATO AECTP Leaflet 2410[3], field data must be acquired based upon the anticipated mission scenario of the unit under test (UUT). As detailed in paragraph 4.4.1.1 and reference number 1, transducer placement and orientation are critical and must be thoroughly documented.

5.1 Reference SDM Development.

As stated in paragraph 4.5, a SDM in terms of the desired rigid body modes to be tested should be computed for each test configuration identified in the mission scenario.

5.1.1 SDM Ensemble CSD Characteristics.

Based on the characteristics of the CSD terms of the ensemble of SDMs, the VSD process will yield a vibration specification consistent with one of the three cases that follow:

a. Case 1. Coherence Terms Approaching Zero (Independent Motion DOFs) – This is the easiest situation to deal with in that each motion DOF ASD may be addressed individually via the same techniques employed in 1-DOF VSD as discussed in reference number 3. When programming the vibration control system, it is recommended that coherence be set to a low non-zero level (i.e. $\gamma^2 = 0.1$) over the test bandwidth of interest. For such a small coherence, the phase parameter is essentially a random variable and establishing a phase specification is not required.

   A special situation that may lead an analyst to develop a MDOF vibration specification with independent motion DOFs, would be a composite specification that encompasses multiple vehicles (i.e. a composite wheeled vehicle specifications comparable to those in MIL-STD-810H, Method 514.8). As each vehicle will tend to have its own CSD characteristics, it is not possible to define CSD terms such that a single coherence and phase relationship addresses each vehicle. Enveloping techniques that work well in addressing magnitude based ASD terms are simply not applicable in addressing phase relationships between mechanical DOFs.

b. Case 2.Non-Zero Coherence across a Portion of the Test Bandwidth – When developing a MDOF vibration specification based on a single platform, one would expect the CSD terms measured across the range of scenarios addressed in the mission scenario to be similar in nature. The dynamic characteristics of the structure and often the proximity of the measurement transducers will greatly influence the CSD characteristics. There are often situations in which coherence between motion DOFs are high and phase is well defined, but only over a portion of the test spectrum. This is a common observation on many wheeled vehicles where coherence is high at lower frequencies (i.e. frequencies below 50 Hertz (Hz) and near zero at higher frequencies. In such scenarios, one would only establish coherence and phase specifications for the portion of the spectrum with high coherence. The remainder of the spectrum would be treated as in Case 1. Also, in establishing CSD reference criteria, the analyst must ensure the resulting criteria is physically realizable (refer to paragraph 4.5.3 for additional detail).

c. Case 3. Non-Zero Coherence across the Full Test Bandwidth – This scenario is comparable to Case 2 with coherence being defined across the entire test bandwidth. It is anticipated that this would be the least likely scenario in a MDOF VSD effort. However, it is also the configuration that will be the most difficult to deal with from both a VSD development aspect and from an implementation perspective. In addition to the issue of ensuring the resulting SDM reference is physically realizable, the classic problem of mechanical impedance mismatch between field and laboratory are often major concerns in implementing a fully defined SDM reference criterion for a laboratory test. Specifically, if the mechanical impedance between field and laboratory are not very well matched (and they usually are not), there may be portions of the spectrum in which coherence may be significantly different than specified and simply not controllable. While this situation is also possible in Case 2, it is almost certain to be an issue in a scenario such as Case 3, in which the entire test bandwidth has a CSD reference criteria. This topic of uncontrollable coherence associated with mechanical impedance mismatches is a control issue for all three Cases and is discussed further in the minimum drive consideration of paragraph 4.5.6.

d. Regardless of which of the three cases the SDM is characterized by, it is highly likely that there will be mechanical impedance differences between the field and laboratory conditions. In some cases these impedance differences may result in excessive drive signals. Although the various control system vendors address this situation in varying degrees, it may still be necessary to address this issue through test operator intervention via techniques such as those identified in paragraphs 4.5.4 and 4.5.5.

5.2 Test Tolerance Recommendations.

Setting tolerances for a MIMO test is challenging given the large amount of information encompassed by the reference autospectra and cross spectra involved. Additionally, the overall energy is not necessarily distributed evenly about each mechanical DOF and dominant DOFs often tend to dominate the control. The objective here is to establish a reasonable starting point in establishing test tolerances. Experience with specific test configurations may be employed to refine the basic guidance defined below. As usual, any test specific test tolerances should be clearly documented within the test plan.

a. Autospectra(1): ±3 dB for $f \le 500Hz$ and ±6dB for $f > 500Hz$.

(1)The portion of the spectrum that actually reaches the maximum tolerance limits is anticipated in narrow bandwidths. The tolerance on the overall Grms level of each controlled DOF shall be within ±15% of the corresponding reference.

b. Cross spectra: Define tolerances in terms of Phase and Coherence. Note that there will be a statistical variation of coherence and phase estimates as a function of the statistical DOFs used to estimate the control SDM and also as a function of the coherence between inputs. Take caution in that the expected statistical variation imposes a lower limit on how tight the respective tolerance can be. The coherence and phase guidance given below should be used in the absence of tolerances determined from field environments and necessary margin needed to account for laboratory and in-field service mechanical impedance mismatches. Note that there may be scenarios in which coherence and phase tolerances need not be defined for the entire test bandwidth.[19]

(1) Coherence: For ordinary coherence in the range $0.5 \le \gamma^2 < 1.0$, set the tolerance to be ±0.1 (avoid establishing a coherence reference or tolerance of 1.0).

(2) Phase: If $\gamma^2 < 0.5$, any phase is acceptable. If $0.5 \le \gamma^2 < 1.0$ and the frequency f is <u>within the band</u> $f_h \pm 3\Delta f$ where f_h is a frequency where the reference rate of phase change is more than $10° / Hz$ and Δf is the line spacing of the reference spectra, the default tolerance on phase will be $\pm 40°$. Otherwise, if outside of a frequency band referenced with such high rates phase change, the default tolerance on phase will be $\pm 10°$.

c. Limiting: See paragraph 4.5.4.

5.3 Laboratory Data.

In the case the reference SDM is directly employed as the reference in a MET test (i.e. input/output (I/O) Transformation Control as discussed in reference number 9), and rigid body presumptions are sound, the control accelerometers are not required to be placed in the exact same location in the laboratory as they were used in the original acquisition phase. The critical parameter is that all control locations employed in the laboratory test are referenced to the same "origin" as selected in the original VSD development. However, it is often desirable, based on making position specific comparisons between field and laboratory data, to match the laboratory control locations to the original measurement points.

6. MDOF VSD METHODS.

6.1 Options Considered.

Having reviewed the data acquisition and analysis requirements, this section is dedicated to defining the steps for two candidate MDOF VSD methodologies[17]. Method I is processed in the SDM domain and Method II conducts averaging steps in the Cholesky Domain. An example follows in paragraph 6.3.

6.1.1 Method I.

The following is a 10 step outline of Method I (SDM Domain) MDOF VSD:

Step 1 Prepare to convert field measurements into motion DOFs.

Identify position vectors $r_1 - r_n$ and row selection vectors e_j as defined in paragraph 4.4.1.1, corresponding to the field measurements.

Identify the mission scenario.

Identify the frequency bandwidth of interest.

Identify the sampling frequency of the field measurements.

Step 2 Transform the field measurements into motion DOF's per equation (4.3) for each "Run" identified in the mission scenario.

Step 3 Compute the SDM for each run identified in Step 2. The dimension of the resulting SDM's will be [6x6xd], where d is the number of spectral lines being considered to addresses the frequency bandwidth of interest.

Since the SDM is computed from measured field data, it should be positive definite; however, a check should be performed to verify that each individual SDM is positive definite. This serves as an excellent data quality check.

Refer to the guidance in Step 7 if minor corrections are required to force an individual SDM to be positive definite.

Step 4 Convert the CSD terms (the off-diagonal terms of the SDM) into a normalized form in which the magnitude squared of the cross terms correlates to the ordinary coherence while leaving the phase unchanged.

This is accomplished by normalizing (dividing) the CSD terms by $\sqrt{G_{xx}G_{yy}}$.

While it is not absolutely necessary to conduct this step, it is often easier to understand the physical meaning of the CSD terms when viewed in terms of phase and coherence.

Step 5 Either organize all of the SDM's for the Runs of interest into a logical structure or merge them into one file of known matrix structure such as [SDM_Run1,SDM_Run2….SDM_RunN] to optimize the conduct of basic statistics.

Step 6 Compute a weighted average SDM of the N SDM's of Step 5.

It is critical that the weighted average be performed on the true complex CSD terms (**not** the normalized SDM).

The weighting factor on the average will be directly correlated to the mission scenario times identified in Step 1. If the individual Runs are positive definite, the resulting average should also be positive definite. However, numerical issues may yield non-positive definite results. To minimize numerical issues, average only the lower triangular portion of the SDM and fill in the upper triangular portion of the SDM by taking advantage of the Hermitian structure of the matrix [16].

Any type of enveloping operation should be avoided as it is highly likely to yield a non-positive definite result.

Step 7 As SDM data are manipulated through activities such as averaging, it is advisable to verify the results remain positive definite. As discussed above, occasional numerical issues may be of concern in some instances. If required, force the SDM computed in Step 6 to be positive definite.

This is done by systematically reducing the magnitude of the cross spectral density terms until the Cholesky decomposition is possible at each depth (spectral line) of the SDM. (If required, this process may be somewhat conservative in its reduction of the coherence between DOFs in that the systematic reduction of cross term magnitudes is applied to each cross term equally).

Step 8 Scale the diagonal terms of the autospectra (the diagonal terms of the SDM) resulting from Step 7 to the maximum rms level of each of the N SDM's in Step 5 on an individual DOF basis using Miner-Palmgren.

Observe that a new total test time will be computed for each DOF and that it is highly probably that the resulting test times for each DOF will not be the same.

Since the magnitude of the autospectra are being increased while not modifying the cross-spectral density terms, the resulting scaled SDM should still be positive definite. However, as discussed in Step 7, it is highly recommended that anytime a SDM is manipulated, it should be verified that the resulting SDM remains positive definite.

Step 9 Review the test time associated with each DOF resulting from Step 8 and select a reasonable test time to which the entire SDM may be referenced to. In this step, avoid scaling the dominant DOF by more than the maximum envelope of measured values for that DOF.

Just as in the case of a 1-DOF VSD development, one should consider the general guidance to keep the final test amplitudes resulting from time compression to be no more than 3 dB above the maximum measured field data. Once a test time is selected, reapply Miner-Palmgren as required per DOF. Again make sure the resulting SDM is positive definite and modify as required per Step 7.

Step 10 Scale the results from Step 9 up by up to 3 dB, while not exceeding 3dB above the envelope of measured values per DOF, to account for uncontrolled variables such as fleet variations and scenario conditions not considered in the mission scenario. There are often practical limitations in maintaining all DOF's within 3 dB of the envelope of measured values from their respective DOF. In such cases, attempt to associate the maximum compression with the lowest level DOF or a DOF known to be mechanically robust. The resulting SDM and the test time association per Step 9 define the final specification.

This is accomplished by pre and post multiplying the SDM by the square root of the ratio of the desired scaling factor as:

$\mathbf{S}_{\mathbf{Ynew}} = \mathbf{S}_s \mathbf{S}_{\mathbf{Yold}} \mathbf{S}_s$ (e.g. to scale the SDM ASD terms by 3 dB while keeping the phase and ordinary coherence the same, the diagonal terms of $\mathbf{S}_s$ would be defined as $S_{s,ii} = \sqrt{2}$).

[In the event the user has documented evidence that the mission scenario is of sufficient fidelity to minimize uncontrolled variables, the default scale factor of 3 dB in this step may be reduced].

6.1.2 Method II.

The following is a 10 step outline of Method II (Cholesky Domain) MDOF VSD:

Step 1-4 Correlate directly to Method I Outline.

Step 5 Perform a Cholesky decomposition on the individual SDM associated with each Run in the mission scenario.

Since each individual Run was based on a physical event, the individual SDM's should be positive definite, thereby making the Cholesky decomposition possible. (Recall all Runs would have been tested to verify each was positive definite or corrected as required per Step 3).

Either organize all of the lower triangular matrices resulting from the Cholesky decomposition for the Runs of interest into a logical structure or merge them into one file of known matrix structure such as [CHOL_Run1,CHOL_Run2….CHOL_RunN] to optimize the conduct basic statistics.

Step 6 Compute a weighted average Lower Triangular Matrix of the N Cholesky decompositions of Step 5.

The weighting factor on the average will be directly correlated to the mission scenario identified in Step 1. Note that the resulting average will still consist of positive eigenvalues implying that when converted back into the SDM format that the result will be positive definite.

Once converted back into the SDM domain, the resulting CSD terms will generally be highly comparable to the average CSD values computed in Step 6 of Method I. However, the rms levels of the ASD terms will not be the same. In addition, the spectral shape of the ASD terms will generally have been slightly modified.

Step 7 Rescale the ASD terms of the SDM resulting from Step 6 to match the rms levels of those in Method 1 Step 6.

Convert the CSD terms (the off-diagonal terms of the SDM) into a normalized form in which the magnitude squared of the cross terms correlates to the ordinary coherence while leaving the phase unchanged. (Again, while it is not absolutely necessary to conduct this step, it is often easier to understand the physical meaning of the CSD terms when viewed in terms of phase and coherence).

Observe that Method II involves the averaging of matrix square roots. The resulting SDM phase and coherence are expected to be very similar to those of the averaged field data produced in Method I. The ASD terms spectral shapes are expected to be slightly different (i.e. < 3 dB per spectral line for SDM's of similar statistical variance). The actual differences depend to a great deal on the statistical variation of the component square roots. If the statistical variation is significant, one may consider developing multiple references by grouping runs with similar spectral shapes or by reverting to Method I.

Step 8-10 Correlate directly to Method I Outline.

6.2 Example.

a. To illustrate the process discussed above, a simple example was derived (Method I is addressed first). Using an available wheeled vehicle, the input to an onboard missile storage rack was instrumented as shown in Figure 4. The transducer at the center of Figure 4 was placed at the user defined origin, position [0,0,0], in terms of a Cartesian coordinate system. In a traditional right hand orientation, the forward direction of the vehicle was defined as the positive x-axis, towards the vehicle driver's side was considered positive y-axis, and through the vehicle roof was considered the positive z-axis. All transducers are referenced in terms of their relative placement to the origin as discussed previously in the acceleration transformation section of this Annex.

Figure 4. Transducer placement (input to missile rack).

b. Method I Example.

(1) Having established a clear coordinate system definition, the key parameters discussed in Step 1 are identified. In distance units of inches, the positions of the four corner accelerometer locations used in this example are defined as:

$r1=[-17,-6,0]',\ r2=[-17,6,0]',\ r3=[17,-6,0]',\ r4=[17,6,0]'$, which in skew symmetric form are:

$$\left[{}^{P}r_1^{P}\right]^x=\begin{bmatrix}0&0&-6\\0&0&17\\6&-17&0\end{bmatrix},\left[{}^{P}r_2^{P}\right]^x=\begin{bmatrix}0&0&6\\0&0&17\\-6&-17&0\end{bmatrix},\left[{}^{P}r_3^{P}\right]^x=\begin{bmatrix}0&0&-6\\0&0&-17\\6&17&0\end{bmatrix},\left[{}^{P}r_4^{P}\right]^x=\begin{bmatrix}0&0&6\\0&0&-17\\-6&17&0\end{bmatrix}$$

For convenience, the instrumentation team placed the tri-axial transducers such that the channel used to measure the y-axis motion was actually 180 degrees out of phase with respect to the referenced coordinate system. This issue is addressed by simply defining row selection vectors as $e_x^T=[1,0,0], e_y^T=[0,-1,0], e_z^T=[0,0,1]$. Matrix $\overline{T}_a$ and matrix T_a may now be computed as per the discussion in paragraph 4.4.1.1 as:

$$\overline{T}_a=\begin{bmatrix}1&0&0&0&0&6\\0&-1&0&0&0&17\\0&0&1&-6&17&0\\1&0&0&0&0&-6\\0&-1&0&0&0&17\\0&0&1&6&17&0\\1&0&0&0&0&6\\0&-1&0&0&0&-17\\0&0&1&-6&-17&0\\1&0&0&0&0&-6\\0&-1&0&0&0&-17\\0&0&1&6&-17&0\end{bmatrix}$$

$$T_a=\begin{bmatrix}0.2500&0&0&0.2500&0&0&0.2500&0&0&0.2500&0&0\\0&-0.2500&0&0&-0.2500&0&0&-0.2500&0&0&-0.2500&0\\0&0&0.2500&0&0&0.2500&0&0&0.2500&0&0&0.2500\\0&0&-0.0417&0&0&0.0417&0&0&-0.0417&0&0&0.0417\\0&0&0.0147&0&0&0.0147&0&0&-0.0147&0&0&-0.0147\\0.0046&0.0131&0&-0.0046&0.0131&0&0.0046&-0.0131&0&-0.0046&-0.0131&0\end{bmatrix}$$

The field data were sampled at 4096 Hz and the bandwidth of interest is 500 Hz. For the example at hand, a mission scenario was established using a Beta distribution as discussed in reference number 3, and is illustrated in Table 1. Allowing for the time associated with speeds below 5 miles per hour (mph), the total mileage represented is approximately 300.

(2) The field data were then converted into motion DOFs, $\{\mathbf{c}\}_{\mathbf{Motion}}$, using Equation 4.3 per Step 2.

(3) The time histories, $\{\mathbf{c}\}_{\mathbf{Motion}}$ were then transformed into the frequency domain in the form of a SDM per run as described in Step 3. Each SDM was tested per the Cholesky decomposition property and verified to be positive definite.

(4) Each SDM was then normalized as suggested in Step 4 to allow the analyst to review the degree of coherence between DOFs.

Table 1. Mission scenario.

Road Classification	Speed (mph)	Time (hrs)	Distance (miles)
Embedded Rock	5	.690	3.45
	10	1.545	15.45
	15	.737	11.05
Cross Country	10	5.18	51.80
	20	6.332	126.64
	30	2.002	60.06
Radial Washboard	5	.811	4.055
	7	1.841	12.88
	10	1.183	11.83

(5) Per Step 5, the SDMs were configured into a convenient structure to allow statistical analysis. The data were configured as SDM_all=[SDM_Run1,SDM_Run2….SDM_Run8] . Observe only 8 of the 9 runs identified in the scenario are being considered. In reviewing the field data, the 5 mph radial washboard data were significantly lower than the rest of the Runs, determined to have no effect on fatigue, and were not considered in computing the basic statistics of the ensemble.

(6) Next, per Step 6, a weighted average in terms of the time per road condition as defined in Table 1 was computed. This average should be computed in terms of complex CSD terms, not the normalized SDM. The resulting weighted average SDM was then tested at each spectral line to establish whether or not the positive definite criterion was met. Figure 5 illustrates the weighted average SDM. Taking advantage of the Hermitian property of a SDM, Figure 5 is laid out such that the lower triangular section represents the phase between DOFs, the upper triangular portion represents the square root of the ordinary coherence, and the diagonal terms are the ASDs of the 6 rigid body DOFs. Although too small to review in detail on a single page as shown, the coherence plots are all scaled between 0.1 and 1.0. This is to illustrate there is some level of coherence, particularly below 100 Hz in the example at hand, between DOFs. Using the VSD process proposed, the analyst will try to keep as much coherence in the final specification as possible while still ensuring the final result is positive definite.

(7) In order to address the possibility of having to deal with non-positive definite results, a utility was written which gradually and equally reduces the magnitudes of the cross spectral density terms until the positive definite criterion is met per Step 7. This technique actually reduces the cross term magnitudes of some CSDs more than what is required. Addressing this potential shortcoming is one of the motivations for the development of Method II.

(8) At this point, per Step 8, the rms level was computed for each ASD (diagonal SDM Entry) over the bandwidth of interest (3-500 Hz in this example). Each ASD was then scaled to the level of the maximum rms level via Equation 4.6.

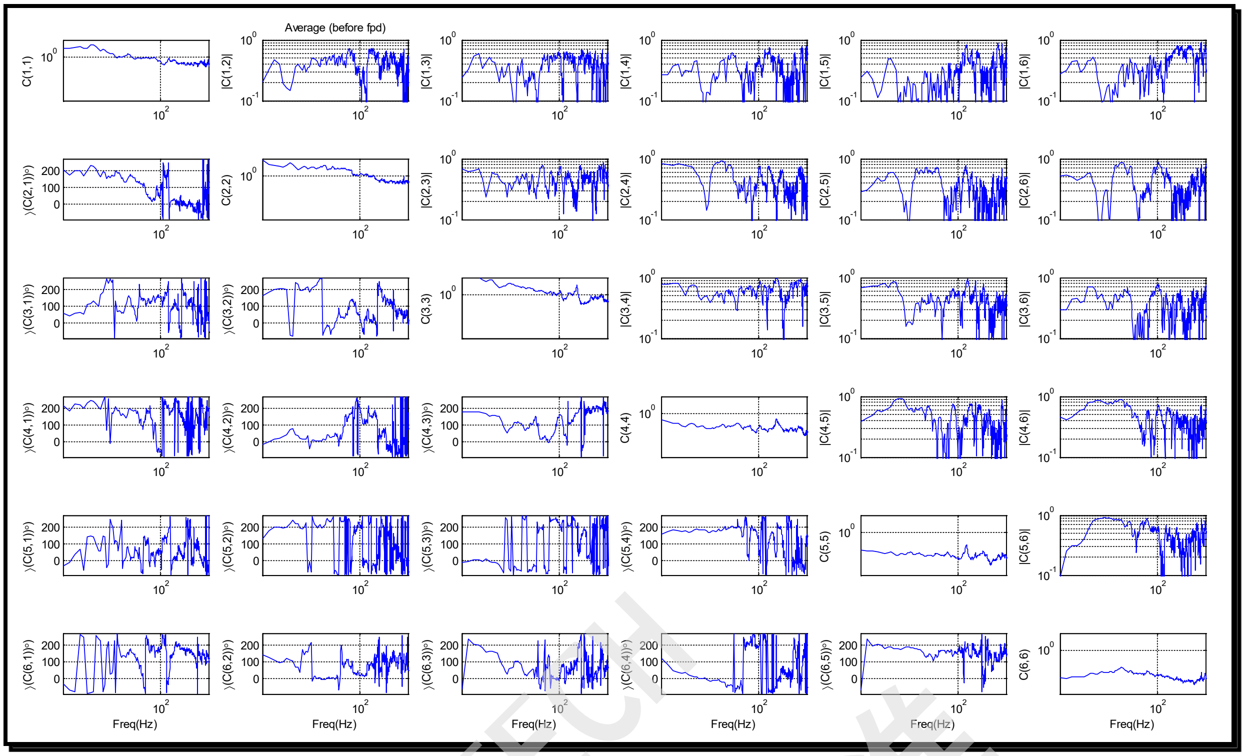


Figure 5. Normalized weighted average SDM.

(9) Per Step 9, the new test times associated with each ASD were also documented. As expected, the new times associated with each DOF were no longer the same. Since the VSD effort is designed to yield a simultaneous 6-DOF reference, it will be necessary to choose a common test time and rescale all ASD entries to the selected test duration. For the example at hand, a test duration of 15 minutes was selected. As is always the case with selection of compressed test durations, one should adhere to the guidance of not exaggerating the ASD power levels by more than 2:1. Of course when dealing with 6 ASD terms, this is not always possible. In such cases, the analyst should avoid increasing the dominant DOFs or DOFs with known structural shortcomings by more than 3 dB above maximum measured ASD levels.

(10) The terms comprising the SDM were based on average ASD and CSD estimates, which is in contrast to the guidance provided in reference number 3, in which the ASD levels carried through the calculations of a 1-DOF VSD were actually based on an ASD computed as the sum of a Mean ASD and standard deviation computed on a per spectral line basis. Working directly with the mean ASD levels is intended to avoid excessive conservatism in the VSD process. Conservatism intended to address uncontrolled variables such as fleet variations and conditions not considered in the mission scenario are addressed by a single scalar (+3 dB in this example) in Step 10. Clearly the analyst has the ability to modify the final conservatism level based on knowledge of the specific VSD effort.

The final reference SDM produced by Method I is shown in Figure 6. Observe that the phase and coherence terms are essentially unchanged from that of the average SDM of Figure 5.

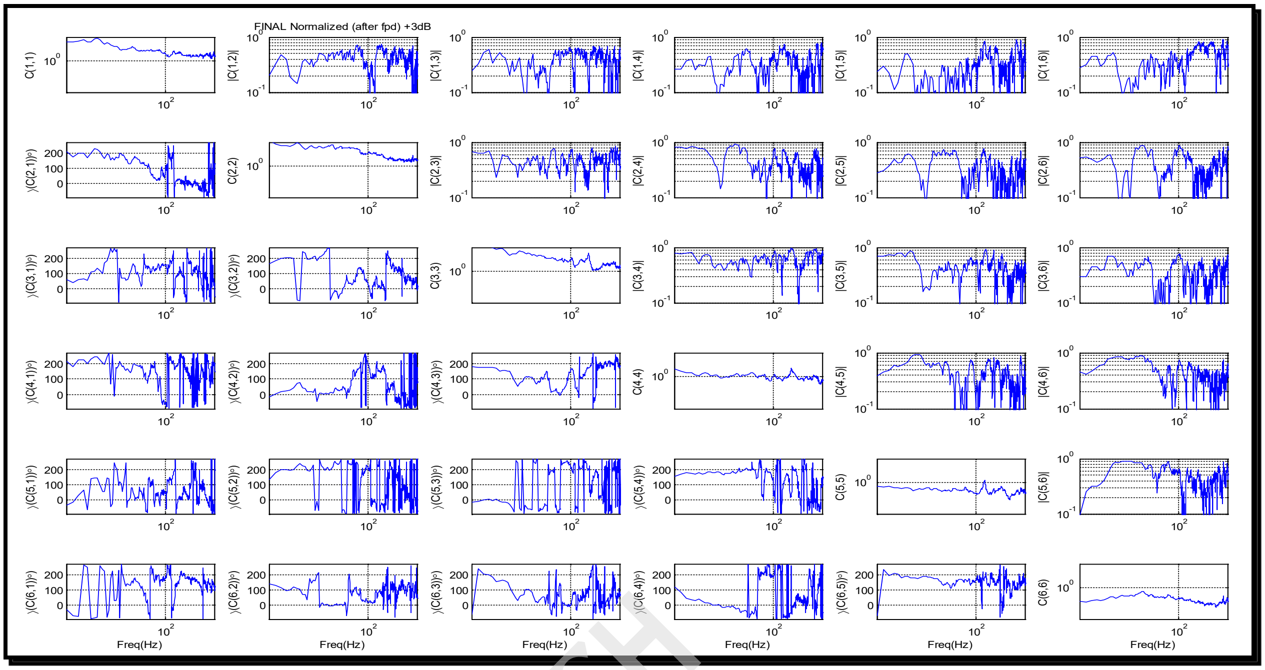


Figure 6. Method I normalized reference SDM.

c. Method II Example. The first four steps of Method II correlate directly to that of Method I. The major deviation in Method II is that all averaging will be computed in the Cholesky domain. In Step 5, Cholesky decompositions are carried out on the individual SDM's associated with each Run in the mission scenario. Since each individual Run was based on a measured physical event, the individual SDMs were positive definite as expected, thereby making the Cholesky decomposition possible. In the event that a given Run had failed the Cholesky decomposition and all measurement locations and relative polarities were verified; investigate the spectral lines at which the decomposition fails. If the decomposition is failing at only a few spectral lines, it may be possible to salvage the measurement employing the CSD magnitude reduction techniques proposed in Method I. The Cholesky domain data were then organized into a convenient structure for statistical analysis. As in Method I, Matlab was used to compute the weighted averages and the Cholesky domain data were organized as: CHOL_all=[CHOL_Run1, CHOL_Run2…..,CHOL_RunN]. In Step 6, a weighted average in terms of the time per road condition as defined in Table 1 was computed over the lower triangular matrix of the eight Cholesky decompositions of Step 5. The weighted average was then converted back into the SDM domain. As expected, the coherence characteristics of the resulting SDM were comparable with that of Figure 5 and the rms levels of the ASD terms required rescaling per Step 7. Steps 8-10 were carried out directly as stated in the Method I outline.

d. The reference SDM resulting from Method II (Figure 7) yielded similar phase and coherence characteristics to that of the reference SDM resulting from Method I (Figure 6). Note that the Method I example took advantage of averaging only the lower triangular CSD terms, avoiding potential numerical issues, thereby not requiring the SDM to be forced positive definite in a manner that would result in lowering the coherence in a more conservative manner than required.

e. ASD Comparisons. Next, the minor spectral shape deviations between the ASD resulting from the two VSD methods discussed will be illustrated. Figures 8 and 9 show the ASD references for the Z axis (vertical) and rotation about Z axis (Rz) respectively, as produced from both VSD methods. The ASD references are superimposed with the raw (unexaggerated) reference data from which the specifications were created. Observe that the ASD shapes envelope the field data without excessive conservatism.

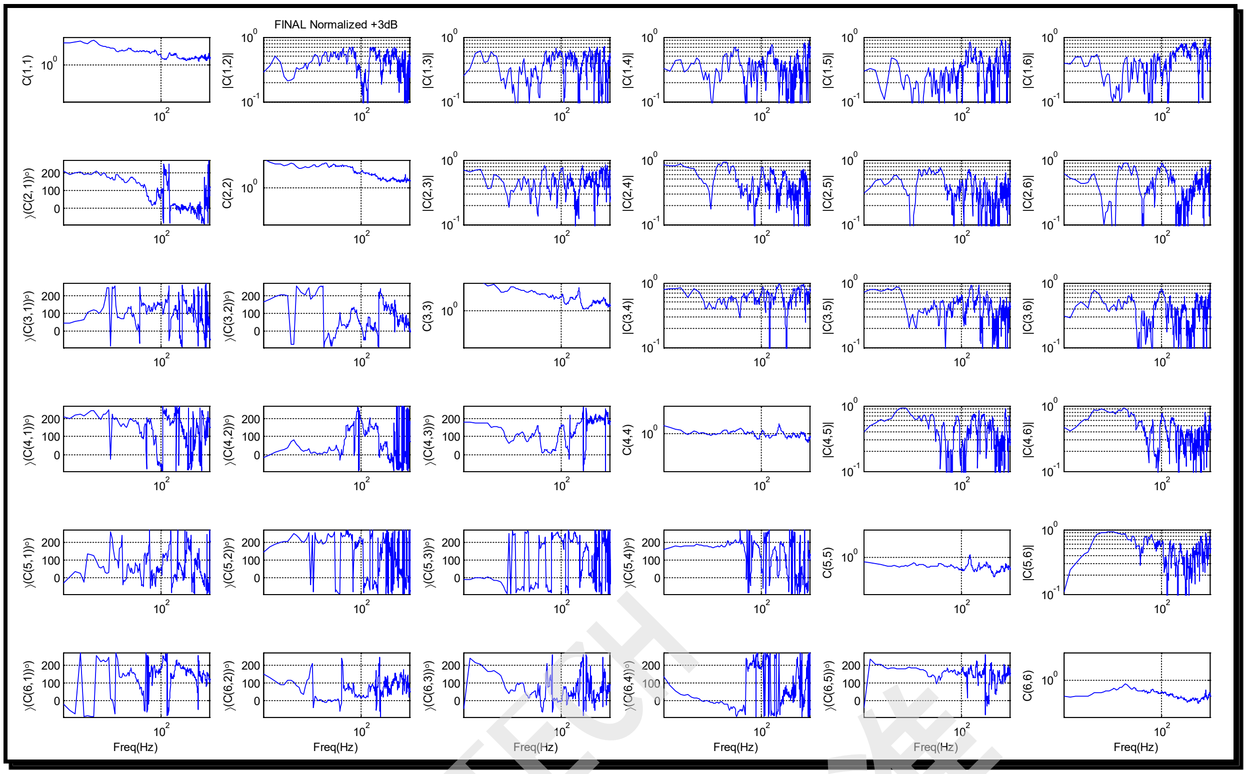


Figure 7. Method II reference SDM.

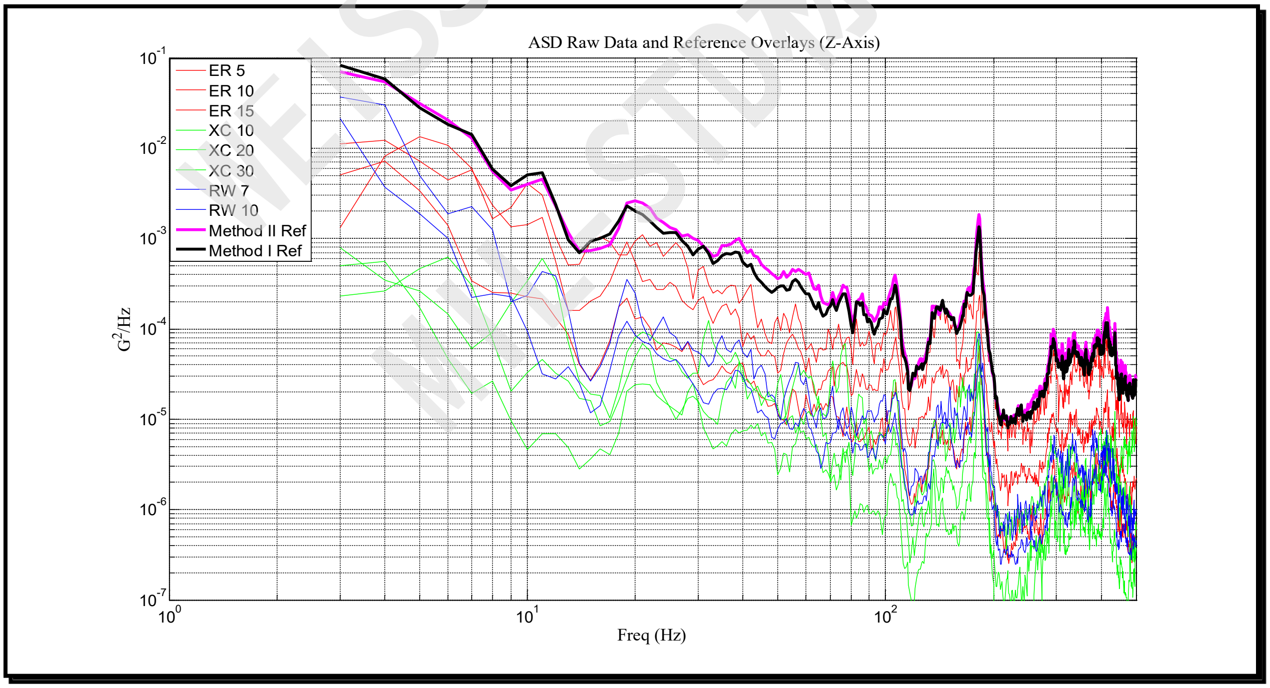


Figure 8. ASD references for the Z axis.

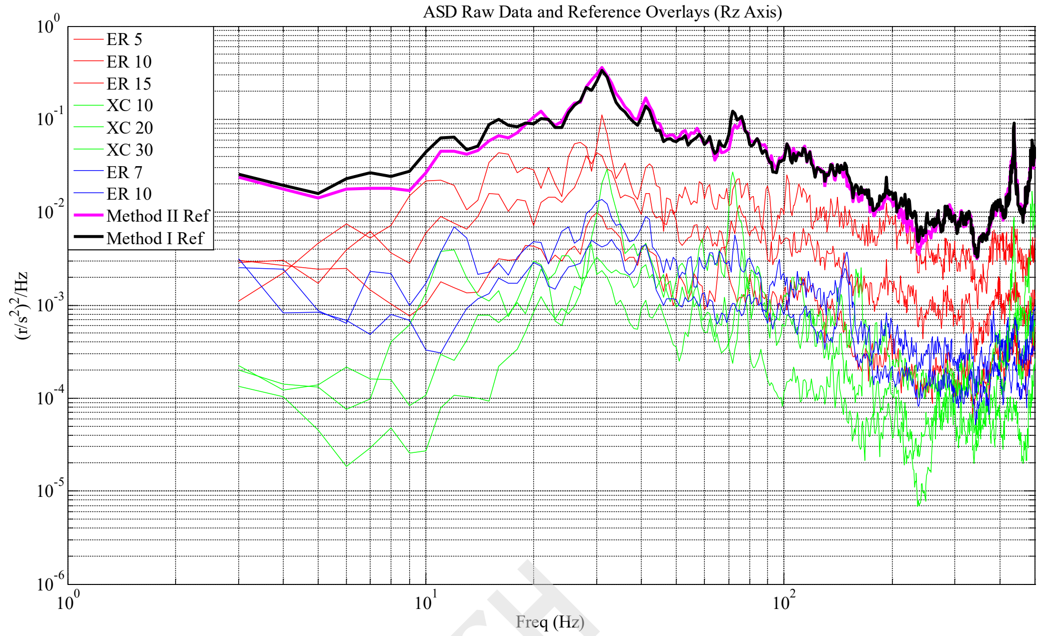


Figure 9. ASD references for rotation about Z axis (Rz).

f. As stated previously, the test duration for the reference SDM yielded by both Methods in this example was established to be 15 minutes. Clearly, as illustrated in Figures 8 and 9 the associated ASD references are highly correlated.

6.3 Concluding Remarks.

a. Two techniques were defined for establishing an input specification for a MDOF system. It was shown that simple enveloping techniques are not appropriate when considering CSD terms due to the sensitivity of such operations associated with maintaining a physically realizable reference. The resulting SDM references yielded through the process outlined are fully populated SDM's. Importing the fully populated SDM into the MDOF control system in an efficient manner is essential due to the volume of information involved.

b. While synthesizing a drive signal with CSD characteristics of the field data is desired, it is recognized that the mechanical impedance of the laboratory configuration is highly unlikely to match that of the field data. Therefore, it will be difficult to maintain CSD characteristics across the spectral bandwidth of interest and thus, the control hierarchy will generally place emphasis on the ASD terms. Also, it is not uncommon in MDOF tests for a specific mechanical degree-of-freedom to consist of a very small percentage of the composite energy across all mechanical degrees-of-freedom. In such cases, the associated error for the low DOF will often be higher than the desired test tolerances and considering global test tolerances may need to be considered.

c. Care was taken in the examples provided to limit the amount of conservatism in the VSD process. One quickly realizes that the amount of conservatism is cumulative across degrees of freedom and if not managed carefully will yield test levels significantly higher than the measured environment. Unlike, the common technique of essentially adding 3 dB to all measurements prior to conducting averaging or enveloping techniques in the 1-DOF arena per reference number 3, all weighted averages in the 6-DOF examples shown were based on raw averaged data. Conservatism to account for variables such as fleet variability and mission scenario omissions were added in the final step. Magnitude amplification associated with time compression techniques was limited to no more than maximum measured levels. Also, on the subject of tolerances, one may find it reasonable to define phase and coherence tolerances over only a portion of the test bandwidth. In the example provided, the coherence dropped off considerably at frequencies above 50 Hz. Since the phase term is essentially a random variable for low coherence, setting tolerances for frequencies greater than 50 Hz would not be recommended for the example shown.

METHOD 527.2, ANNEX E, APPENDIX A

GLOSSARY

Refer to paragraph 1.2.2 of this Method. Additional terms specific to this Appendix follow:

a. **Laboratory Vibration Test Schedule (LVTS)** – All information required to perform a vibration test on a vibration exciter. Information typically includes: a broadband spectra (or profile), sine or narrowband information (if used), test run time, control accelerometer locations, control methods and tolerances, and any test specific information required.

b. **Scenario –** A tabulation of expected exposure events and the corresponding durations.

(This page is intentionally blank.)

METHOD 527.2, ANNEX E, APPENDIX B

ABBREVIATIONS

AECTP	Allied Environmental Conditions Test Publication
ASD	auto spectral density (also referred to as the power spectral density (PSD)
CG	center of gravity
CSD	cross spectral density
dB	decibel
DFT	discrete Fourier transform
DOF	degree of freedom
DTC	US Army Developmental Test Command
FRF	frequency response function
g/V	gravitational units/volts of drive
Hz	hertz
I/O	input/output
IEEE	Institute of Electrical and Electronics Engineers
IES	Institute of Environmental Sciences
IEST	Institute of Environmental Sciences and Technology
LCEP	Life Cycle Environment Profile
LOA	line of action
LVTS	Laboratory Vibration Test Schedule
MA	multi-axis
MDOF	multiple degree-of-freedom
MEMA	multiple-exciter multiple-axis
MESA	multiple-exciter single-axis
MET	multiple exciter test
MIL-STD	Military Standard
MIMO	multiple-input multiple-output
MISO	multiple-input single-output

NATO	North Atlantic Treaty Organization
NBROR	narrowband random on random
pinv	Moore Penrose pseudo inverse
PSD	power spectral density
rms	root mean square
RTC	US Army Redstone Test Center
SA	single-axis
SDM	spectral density matrix
SDOF	single degree-of-freedom
SESA	single-exciter/single-axis
SIMO	single-input multiple-output
SISO	single-input single-output
SOR	sine-on-random
TWR	Time Waveform Replication
UUT	unit under test
VSD	Vibration Schedule Development

METHOD 527.2, ANNEX E, APPENDIX C

NOMENCLATURE

Term	Definition
{ }	A vector where each element is a discrete time history or function of frequency, the discrete Fourier transform (DFT) of a time history. In general lower case letters will be used for functions of time and upper case letters will be used for functions of frequency. Sometimes lower case letters are used to designate an element in a matrix.
[ ]	Will denote a matrix. Usually a third dimension will denote time samples or samples as a function of frequency.
$[\ ]^{T}$	The transpose of a matrix.
$[\ ]'$	The transpose of a real matrix or often used as a compact notation to represent the complex conjugate transpose of a matrix.
$[\ ]^{*\prime}$	The complex conjugate transpose of a matrix (also see $[\ ]'$ above).
$[\ ]^{\dagger}$	The Moore Penrose pseudo inverse of a matrix.
^	Over a variable will denote an estimated value.
{a}	The vector of return acceleration signals.
A	The spectral density matrix of the return signals, typically in units of G^{2}/Hz .
$\{c\}$	A vector of the control signals from a MIMO system. Each element in the vector is a function of time. It can be thought of as a 2 dimensional matrix. First dimension is the input number. The second dimension is the time index.
$\{C\}$	The DFT of $\{c\}$.
C	The spectral density matrix of the control signals. The diagonal elements are the real auto-spectral densities of the control signals. The off diagonal elements are complex functions of frequency giving the cross spectral density between pairs of control signals.
{d}	A vector of drive signals into a MIMO system. Each element in the vector is a function of time. It can be thought of as a 2 dimensional matrix. First dimension is the input number. The second dimension is the time index.
[**D**]	The drive signals in the frequency domain. $\{d\}$ is formed from [D] using a method called time domain randomization. Initially $\mathbf{D} = \mathbf{ZRZ'}$.

Term	Definition
E[ ]	The expected value.
g	The acceleration of gravity.
[**H**]	A matrix of frequency response functions (FRF's) relating the control system response to the drive signals. Typically the elements will have units of g/V. Each element is a frequency response function. A third dimension typically is the amplitude as a function of a set of frequencies relating to the DFT of the input and response signals.
N_s	The number of drive signals, the number of shakers.
N_c	The number of control signals.
N_a	The number of acceleration return signals.
N_d	The number of output control variables.
R	The reference control spectral density matrix; the desired spectral density matrix.
{s}	The vector of shaker drive voltages.
S	The spectral density matrix of the drives in shaker space.
$\mathbf{S}_{CD}$	The spectral density matrix between the control signal and the drives to the shakers.
$\mathbf{T}_a$	The acceleration to control space transformation matrix.
$\mathbf{T}_s$	The drive in the control space to voltages {s} to the shakers transformation matrix.
$\mathbf{Z} = \mathbf{H}^{\dagger}$	The system impedance matrix, typically units of volts/g.

METHOD 527.2, ANNEX E, APPENDIX D

MATRIX ALGEBRA REVIEW

A matrix is an array of numbers arraigned in rows and columns. The size of the matrix is typically stated an [n,m] or n x m, where n is the number of rows and m is the number of columns. In this document 3 dimensional matrices are also used where the third dimension is typically samples in either the time or frequency domain. This Appendix will discuss only two dimensional matrices. It is assumed that if the matrix has 3 dimensions, that the operations can be performed on each 2 dimensional matrix along the third dimension. For example if the matrix is a matrix of frequency response functions, matrix operations will be performed at each frequency line. The definitions provided in this appendix are based on information provided primarily in reference numbers 12 and 13.

a. SDM: A spectral density matrix is a 3 dimensional matrix. At each frequency line (the 3rd index) the matrix is a square complex matrix. Each diagonal element is the autospectrum of the corresponding element. Loosely an element in the SDM is defined as:

$$G_{ji}(k) = 2\lim_{T\to\infty}\frac{1}{T}\mathrm{E}[X_j(k,T)X_i^*(k,T)]$$

where: $G_{ji}(k)$ is the cross spectral density between the j'th and i'th random processes.

$X_j(k,T)$ and $X_i\ (k,T)$ are the discrete Fourier transforms of the time histories, and k is the frequency index. If i = j, the spectrum is called the autospectrum or the power spectrum. In reality, the true spectral density is generally not known and an estimate is employed. Some authors define the elements as:

$$G_{ij}(k) = 2\lim_{T\to\infty}\frac{1}{T}\mathrm{E}[X_x^*(k,T)X_j(k,T)]$$

The SDM matrix is Hermitian positive definite.

b. Hermitian Matrix: A matrix, A, is Hermitian if the diagonal elements are real positive numbers and the corresponding off diagonal elements are complex conjugate pairs:

$$a_{ii} = \text{ positive real number}$$
$$a_{ji} = a_{ij}^* = conj(a_{ij})$$

where: a_{ji} is the element form j'th row, i'th column of A.

Note: All valid spectral density matrices (SDM) are Hermitian.

c. Positive Definite Matrix and Positive Semi-Definite Matrix: If a square Hermitian matrix, A, has all positive eigenvalues, the matrix is positive definite. If the matrix has zero eigenvalues the matrix is positive semi-definite. A Cholesky decomposition is possible for all positive definite matrices.

$$\mathbf{A} = \mathbf{LL'}$$

where: **L** is a lower triangular matrix with real positive values on the diagonal. **L'** is the complex conjugate transpose of **L**. If the matrix, **A**, is positive semi-definite, special care must be taken in computing **L**. If a zero element is found on the diagonal of **L**, the entire column must be set to zero. Computing the Cholesky decomposition is actually the easiest way to check for positive definite. If the algorithm fails the matrix, **A** is not positive definite.

d. Transformation of a Positive Definite Matrix:

$$\text{Let } \mathbf{B} = \mathbf{HAH'}$$

If the matrix A is positive definite, B is positive definite.

Note: All valid SDMs are positive semi-definite or positive definite. Because some noise is always present in measured data, a measured SDM will always be positive definite.

e. Ordinary Coherence, γ^2: The ordinary coherence between two signals is defined as:

$$\gamma_{12}^2 = \frac{|G_{12}|^2}{G_{11}G_{22}}$$

G_{12} is the cross spectral density between the signals and G_{11} and G_{22} are the two autospectra.

The ordinary coherence is bounded by $0 \le \gamma_{12}^2 \le 1$.

Coherence is a measure of the linear relationship between the signals. If the coherence is unity, a perfect linear relationship exists between the signals. If the coherence is zero, the signals are said to be independent, and there is no linear relationship between the signals.

If one or more of the ordinary coherences in a SDM are in unity at any frequency, the matrix is positive semi-definite at that frequency.

f. Singular Value Decomposition: Singular value decomposition has several applications in MIMO testing. Singular value decomposition is defined as:

$$\mathbf{M}=\mathbf{USV'}$$

M is any matrix. **U** and **V'** are orthonormal. This implies that:

$$\mathbf{UU'}=\mathbf{I} \text{ and } \mathbf{VV'}=\mathbf{I}$$

S is a diagonal matrix of non-negative real numbers. A common convention is to order the diagonal elements of **S** in a non-increasing fashion.

g. Pseudo inverse: The Moore Penrose pseudo inverse is used often in MIMO control. Some of the properties are discussed below. The Moore Penrose pseudo inverse can be derived as follows:

$$\mathbf{M} = \mathbf{USV'}$$

$$\mathbf{U'M} = \mathbf{U'USV'} = \mathbf{SV'}$$

$$\mathbf{S}^{-1}\mathbf{U'M} = \mathbf{S}^{-1}\mathbf{SV'} = \mathbf{V'}$$

$$\mathbf{VS}^{-1}\mathbf{U'M} = \mathbf{VV'} = \mathbf{I}$$

$\mathbf{M}^{\dagger} = \mathbf{VS}^{-1}\mathbf{U'}$ is known as the pseudo inverse of **M**.

The inverse of the reduced **S** is easy since the matrix is diagonal. To compute $\mathbf{S}^{-1}$ the elements greater than a tolerance are inverted and kept, the elements less than a tolerance are replaced by zero.

$$\mathbf{MM}^{\dagger}\mathbf{M} = \mathbf{M} \text{ and } \mathbf{M}^{\dagger}\mathbf{M}\,\mathbf{M}^{\dagger} = \mathbf{M}^{\dagger}$$

$\mathbf{MM}^{\dagger}$ and $\mathbf{M}^{\dagger}\mathbf{M}$ are Hermitian

If the number of columns in **M** exceed the number of rows and the rows are independent $\mathbf{MM}^{\dagger}=\mathbf{I}$. If the number of rows in **M** exceeds the number of columns and the columns are independent $\mathbf{M}^{\dagger}\mathbf{M}=\mathbf{I}$. For a more complete discussion see the help file for pinv in MATLAB.

h. Matrix Rank: The rank of a matrix, M, equals the number of non-zero singular values in M. In numerical linear algebra, the singular values can be used to determine the effective rank of a matrix. Define a measure of singular values as the ratio of the singular values and the largest singular value. Let r be the number values greater than a threshold. Where the measure is less than the threshold, set the singular values to zero. The number of non-zero singular values in the resulting matrix is the effective rank of the matrix. The effective rank of the matrix is r. For a square matrix, if r is less than the number of rows and columns in the matrix, the matrix is said to be ill conditioned.

i. Matrix Approximation: Let $\widetilde{M} = usv'$

where: **s** = a diagonal matrix of the singular values greater than a threshold defined as the ratio of the singular values divided by the largest singular value. Let n = the number of kept singular values. **s** has n rows and columns. **u** is the first n columns of **U**. **v'** is the first n rows of **V'**.

$$\widetilde{M} \text{ minimizes } \left\| S - usv' \right\|_F$$

Hence, $\widetilde{M}$ is a very useful approximation of **M**.

j. Frobenius Norm: The Frobenius Norm of matrix M is defined as:

$$\left\| M \right\|_f = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \left| m_{ij} \right|^2} = \sqrt{trace(A'A)} = \sqrt{\sum_{i=1}^{\min(m,n)} \sigma_i^2}$$

where: σ_i are the singular values of M.

k. Trace: The trace of a positive definite matrix is defined as the sum of the diagonal elements. An important property of the trace often of use is:

$$\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$$

l. Rescaling the Autospectra: When generating a SDM it might sometimes be useful to rescale the autospectra and be assured that the result remains positive definite. This can be accomplished by pre and post multiplying by a diagonal matrix of scaling factors. The triple product will rescale the autospectra while keeping the coherence and phase between pairs of channels unchanged.

$$\mathbf{G}_{\mathbf{new}} = \mathbf{S}\mathbf{G}_{\mathbf{old}}\mathbf{S}'$$

where: $\mathbf{G}_{\mathbf{new}}$ is the new positive definite SDM, $\mathbf{G}_{\mathbf{old}}$ is the original positive definite SDM, and $\mathbf{S}$ is a diagonal matrix of scaling factors. Each autospectra will be scaled by the corresponding element in $\mathbf{S}^2$.

This is a convenient way to generate the normalized SDM (the diagonal elements are the autospectra and the magnitude squared of the off diagonal terms are the ordinary coherence and the phase is the phase of the cross spectra). The normalized form is computed by rescaling the SDM to unity autospectra by pre and post multiplying the SDM by a diagonal matrix whose terms are the inverse square root of the autospectra. The resulting unity autospectra are then replaced by the original autospectra.

The inverse is computed by replacing the diagonal autospectra by ones and then rescaling by pre and post multiplying by a diagonal matrix whose terms are the square root of the original autospectra.

m. Kinematic Transformation (Frequency Domain): The time domain based kinematic transformation derived in paragraph 4.4.1 of this Annex yielded:

$$\underset{(n\times 1)}{\{\mathbf{a}\}_{\mathbf{Meas}}} = \underset{(n\times 6)}{\left[\overline{\mathbf{T}}_{\mathrm{a}}\right]} \underset{(6\times 1)}{\{\mathbf{c}\}_{\mathbf{Motion}}} \quad \text{and} \quad \underset{(6x1)}{\{\mathbf{c}\}_{\mathbf{Motion}}} = \underset{(6xn)}{\left[\mathbf{T}_{\mathrm{a}}\right]} \underset{(nx1)}{\{\mathbf{a}\}_{\mathbf{Meas}}}$$

Given the same rigid body constraint, the same input transformation matrix $\left[\mathbf{T}_{\mathbf{a}}\right]$ may be employed to transform a spectral density matrix between motion and measurement space as follows (Paragraph 6.1, reference q):

$$\underset{(nxnxm)}{\left[\mathbf{SDM}_{\mathbf{meas}}\right]} = \underset{(n\times 6)}{\left[\overline{\mathbf{T}}_{\mathrm{a}}\right]} \underset{(6x6xm)}{\left[\mathbf{SDM}_{\mathbf{motion}}\right]} \underset{(6\times n)}{\left[\overline{\mathbf{T}}_{\mathrm{a}}\right]'} \quad \text{and} \quad \underset{(6x6xm)}{\left[\mathbf{SDM}_{\mathbf{motion}}\right]} = \underset{(6\times n)}{\left[\mathbf{T}_{\mathbf{a}}\right]} \underset{(nxnxm)}{\left[\mathbf{SDM}_{\mathbf{meas}}\right]} \underset{(n\times 6)}{\left[\mathbf{T}_{\mathbf{a}}\right]'}$$

n. Proof 1:

An element in $\boldsymbol{S_Y}$ is given by, where n = number of inputs, and m = number of outputs

$$Y_{ij}=\sum_{r=1}^{n}\sum_{k=1}^{n}X_{rk}H_{ir}H_{jr}^{*}\qquad i=1:m\quad j=1:m$$

A diagonal element is given by:

$$Y_{ii}=\sum_{r=1}^{n}\sum_{k=1}^{n}X_{rk}H_{ir}H_{ir}^{*}=\sum_{r=1}^{n}\sum_{k=1}^{n}X_{rk}\,|\,H_{ir}\,|^{2}\quad i=1:m$$

If $\mathbf{S_x}$ is diagonal, $X_{rk}=0,$ if $r\neq k$, then:

$$Y_{ii}=\sum_{r=1}^{n}X_{rr}\,|\,H_{ir}\,|^{2}\qquad i=1:m$$

This can be written as a set of linear equations:

$$\overline{\mathbf{Y}}=\overline{\mathbf{HX}}$$

Which can be solved for $\overline{\mathbf{X}}$ as: $\overline{\mathbf{X}}=\overline{\mathbf{H}}^{-1}\overline{\mathbf{Y}}$

METHOD 527.2, ANNEX E, APPENDIX E

REFERENCES

1. MIL-STD-810G, Department of Defense Test Method Standard: Environmental Engineering Considerations and Laboratory Tests, 31October 2008.

2. Handbook for Dynamic Data Acquisition and Analysis, IEST-RP-DTE012.2, Institute of Environmental Sciences and Technology, Arlington Place One, 2340 S. Arlington Heights Road, Suite 100, Arlington Heights, IL 60005-4516.

3. AECTP 240, Edition 4, Leaflet 2410, Development of Laboratory Vibration Test Schedules, 1 June 2009.

4. Smallwood, D.O., "Multiple Shaker Random Control with Cross Coupling", Proceedings of the Institute of Environmental Sciences (IES), April 18-20 1978, pages 341-347.

5. Underwood, M., "Multi-Exciter Testing Applications: Theory and Practice"; Proceedings of the Environmental Sciences and Technology, ESTECH2002, May 1, 2002, Anaheim, CA.

6. US Army Developmental Test Command (DTC) / US Army Redstone Test Center (RTC) Methodology Study of MIMO Vibration Specification Guidance, March 2010.

7. Welch, P.D., "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Base on Time Averaging Over Short, Modified Periodograms", Institute of Electrical and Electronics Engineers (IEEE) Transactions on Audio and Electroacoustics, Volume AU-15, Number 2 / June 1967.

8. Draft Multi-Shaker Test and Control: Design, Test, and Evaluation Division Recommended Practice 022.1, IEST-RP-DTE022.1, April 2008.

9. Underwood, M. and Keller, T. "Applying Coordinate Transformations to Multi-DOF Shaker Control." Sound and Vibration Magazine, January 2006.

10. Fitz-Coy, N, Hale, M. and Nagabhushan, V., "Benefits and Challenges of Over-Actuated Excitation Systems", Shock and Vibration Journal, Volume 17, Number 3 / 2010.

11. Hale, M and Fitz-Coy, N., "On the Use of Linear Accelerometers in Six-DOF Laboratory Motion Replication: A Unified Time-Domain Analysis", 76th Shock and Vibration Symposium, 2005.

12. Golub and van Loan, Matrix Computations, John Hopkins Press, 3rd Edition, 1989.

13. Bendat, J. and Piersol, A. G., Random Data Analysis and Measurement Procedures, John Wiley and Sons, Inc, New York, 3rd Edition, 2000.

14. Smallwood, D.O., "Multiple-Input Multiple-Output (MIMO) Linear Systems Extreme Inputs/Outputs", Shock and Vibration Journal, Vol. 14, Number 2 / 2007, pages 107-132.

15. Smallwood, D.O., "A Proposed Method To Generate a Spectral Density Matrix for a Multiple Input, Multiple Output (MIMO) Vibration Test", 80th Shock and Vibration Symposium, 2010.

16. Wirsching, Paez, and Ortiz, Random Vibrations, Theory and Practice, Wiley 1995.

17. Hale, M., "A 6-DOF Vibration Specification Development Methodology", Journal of the IEST, V. 54, No. 2, 2011.

18. Hale, M., "Spectral Density Matrix Transformations", Journal of the IEST, V. 60, No. 1, pp 17-26, 2017.

19. Martin, L and Schneider, S, "On the Nature of Spectral Density Matrices used to Characterize Vibration Environments", Proceedings of the 87th Shock and Vibration Symposium, Oct 2016, pp 349-363.

(This page is intentionally blank.)